

Solids of Revolution Worksheet**Part I**

1. Sketch the region bounded by the curves $y = x^4$, $y = 2 - x$ and $y = 0$.

If you revolve this region to create a solid, what variable of integration should you use to compute the volume of that solid?

- a. If you revolved this region around the x -axis, what method should you use to compute the volume of the resulting solid?
 - b. If you revolved this region around the y -axis, what method should you use?
 - c. If you revolved this region around the line $x = 2$, what method should you use?
 - d. If you revolved this region around the line $y = 1$, what method should you use?
2. Sketch the region bounded by the curves $y = e^x$, $y = x + 2$ and $x \geq 0$.

If you revolve this region to create a solid, what variable of integration should you use to compute the volume of that solid?

- a. If you revolved this region around the x -axis, what method should you use to compute the volume of the resulting solid?
- b. If you revolved this region around the y -axis, what method should you use?
- c. If you revolved this region around the line $y = 1$, what method should you use?
- d. If you revolved this region around the line $x = 3$, what method should you use?

Part II

Sketch the region bounded by the curves $y = 3x$, $y = 12 - 3x$, and $y = 0$.
Label all the intersection points.

Variable of integration: _____

1. a. Sketch a picture of the region revolved around the x-axis.

- b. What's best method to compute the volume of the solid?
Fill in the chart for that method, and compute the volume.
(You only have to fill in one column of the chart.)

Disks/Washers	Shells
<u>Inner Radius (I.R.):</u>	<u>Radius:</u>
<u>Outer Radius (O.R.):</u>	<u>Height:</u>
$\int_{\underline{\quad}}^{\underline{\quad}} \pi [O.R.]^2 d_{\underline{\quad}} - \int_{\underline{\quad}}^{\underline{\quad}} \pi [I.R.]^2 d_{\underline{\quad}} :$	$\int_{\underline{\quad}}^{\underline{\quad}} 2\pi (Radius)(Height) d_{\underline{\quad}} :$

Volume = _____

2. a. Sketch a picture of the region revolved around the y-axis.

b. Choose a method to find the volume, fill in the chart for that method, and compute the volume.

Disks/Washers	Shells
<u>Inner Radius (I.R.):</u>	<u>Radius:</u>
<u>Outer Radius (O.R.):</u>	<u>Height:</u>
<u>$\int_{-}^{+} \pi [O.R.]^2 d_{-} - \int_{-}^{+} \pi [I.R.]^2 d_{-} :$</u>	<u>$\int_{-}^{+} 2\pi (Radius) (Height) d_{-} :$</u>

Volume = _____

3. a. Sketch a picture of the region revolved around the line $x = 5$.

b. Choose a method to find the volume, fill in the chart for that method, and compute the volume.

Disks/Washers	Shells
<u>Inner Radius (I.R.):</u>	<u>Radius:</u>
<u>Outer Radius (O.R.):</u>	<u>Height:</u>
<u>$\int_{-}^{+} \pi [O.R.]^2 d_{-} - \int_{-}^{+} \pi [I.R.]^2 d_{-} :$</u>	<u>$\int_{-}^{+} 2\pi (Radius) (Height) d_{-} :$</u>

Volume = _____

4. a. Sketch a picture of the region revolved around the line $y = -1$.

b. Choose a method to find the volume, fill in the chart for that method, and compute the volume.

Disks/Washers	Shells
<u>Inner Radius (I.R.):</u>	<u>Radius:</u>
<u>Outer Radius (O.R.):</u>	<u>Height:</u>
<u>$\int_{-}^{-} \pi [O.R.]^2 d_{-} - \int_{-}^{-} \pi [I.R.]^2 d_{-} :$</u>	<u>$\int_{-}^{-} 2\pi (Radius) (Height) d_{-} :$</u>

Volume = _____

Part III

Sketch the region bounded by the curves $y = x$ and $y = x^2 - 4x + 4$.
Label all the intersection points.

Variable of integration: _____

1. a. Sketch a picture of the region revolved around the x-axis.

- c. Choose a method to compute the volume of the solid and fill in the chart for that method.

Disks/Washers	Shells
<u>Inner Radius (I.R.):</u>	<u>Radius:</u>
<u>Outer Radius (O.R.):</u>	<u>Height:</u>
<u>$\int_{-}^{+} \pi [O.R.]^2 d_{-} - \int_{-}^{+} \pi [I.R.]^2 d_{-} :$</u>	<u>$\int_{-}^{+} 2\pi (Radius) (Height) d_{-} :$</u>

2. a. Sketch a picture of the region revolved around the y-axis.

- b. Choose a method to compute the volume of the solid and fill in the chart for that method.

Disks/Washers	Shells
<u>Inner Radius (I.R.):</u>	<u>Radius:</u>
<u>Outer Radius (O.R.):</u>	<u>Height:</u>
<u>$\int_{-}^{+} \pi [O.R.]^2 d_{-} - \int_{-}^{+} \pi [I.R.]^2 d_{-} :$</u>	<u>$\int_{-}^{+} 2\pi (Radius) (Height) d_{-} :$</u>

3. a. Sketch a picture of the region revolved around the line $y = 4$.

- b. Choose a method to compute the volume of the solid and fill in the chart for that method.

Disks/Washers	Shells
<u>Inner Radius (I.R.):</u>	<u>Radius:</u>
<u>Outer Radius (O.R.):</u>	<u>Height:</u>
<u>$\int_{-}^{-} \pi [O.R.]^2 d_{-} - \int_{-}^{-} \pi [I.R.]^2 d_{-} :$</u>	<u>$\int_{-}^{-} 2\pi (Radius)(Height) d_{-} :$</u>

4. a. Sketch a picture of the region revolved around the line $x = 1$.

- b. Choose a method to compute the volume of the solid and fill in the chart for that method.

Disks/Washers	Shells
<u>Inner Radius (I.R.):</u>	<u>Radius:</u>
<u>Outer Radius (O.R.):</u>	<u>Height:</u>
<u>$\int_{-}^{-} \pi [O.R.]^2 d_{-} - \int_{-}^{-} \pi [I.R.]^2 d_{-} :$</u>	<u>$\int_{-}^{-} 2\pi (Radius)(Height) d_{-} :$</u>

Answers to Solids of Revolution Worksheet

Part I

- Variable of integration: y
 - Shells
 - Disks/Washers
 - Disks/Washers
 - Shells
- Variable of integration: x
 - Disks/Washers
 - Shells
 - Disks/Washers
 - Shells

Part II

Variable of integration: y

1. (Shells) $\int_0^6 2\pi\left(4 - \frac{2}{3}y\right) dy$

Volume: 48π

2. (Disks/Washers) $\int_0^6 \pi\left(4 - \frac{1}{3}y\right)^2 dy - \int_0^6 \pi\left(\frac{1}{3}y\right)^2 dy$

Volume: 48π

3. (Disks/Washers) $\int_0^6 \pi\left(5 - \frac{1}{3}y\right)^2 dy - \int_0^6 \pi\left(1 + \frac{1}{3}y\right)^2 dy$

Volume: 72π

4. (Shells) $\int_0^6 2\pi(y+1)\left(4 - \frac{2}{3}y\right) dy$

Volume: 72π

Part III

Variable of integration: x

1. (Disks/Washers) $\int_1^4 \pi(x)^2 dx - \int_1^4 \pi(x^2 - 4x + 4)^2 dx$

2. (Shells) $\int_1^4 2\pi x(-x^2 + 3x - 4) dx$

3. (Disks/Washers) $\int_1^4 \pi(-x^2 + 4x)^2 dx - \int_1^4 \pi(4 - x)^2 dx$

4. (Shells) $\int_1^4 2\pi(x-1)(-x^2 + 3x - 4) dx$