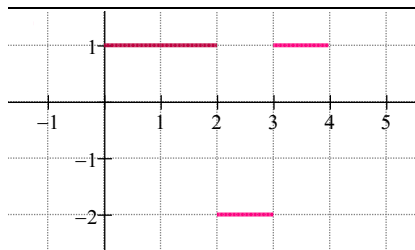


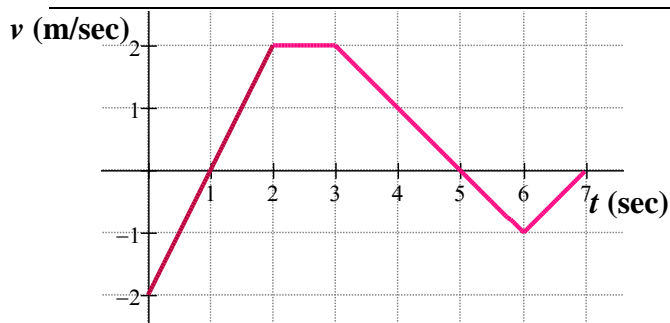


More Motion Problems

1. An object moving on a horizontal line has velocity $v(t) = 5 \cos t$ mph in the time interval $0 \leq t \leq 2\pi$ hours.
- Find the time subintervals in which the object moves to the right, and those in which it moves to the left.
 - Set up integral expressions to find the distance the object travels in each of these subintervals. Evaluate those expressions **without using your calculator**.
 - Use your answer to part (b) to find the displacement and the total distance traveled in the time interval $0 \leq t \leq 2\pi$ hours.
 - Set up a single integral to find the displacement in the time interval $0 \leq t \leq 2\pi$ hours. **Use your calculator to evaluate your integral.**
 - Set up a single integral to find the total distance traveled in the time interval $0 \leq t \leq 2\pi$ hours. **Use your calculator to evaluate your integral.**
-
2. An object moves on a vertical line in the time interval $0 \leq t \leq 16$ seconds, such that its acceleration is $a(t) = \sqrt{t}$ (ft/sec)/sec and at time $t = 0$, $v(0) = -18$ ft/sec.
- Find a formula for the velocity of the object at any time t .
 - Set up an integral to find the displacement in the time interval $0 \leq t \leq 16$ seconds. **Use your calculator to evaluate your integral.**
 - Set up an integral to find the total distance traveled in the time interval $0 \leq t \leq 16$ seconds. **Use your calculator to evaluate your integral.**
-
3. A particle follows a linear motion in the time interval $[0, \frac{\pi}{2}]$, where time is measured in hours. The position of the particle is given by $s(t) = 8 \sin t + 9$, where distances are measured in kilometers.
- Find the displacement in the time interval $[0, \frac{\pi}{2}]$ hours.
 - Find a formula for the velocity of the object at any time t .
 - Set up an integral to find the total distance traveled in the time interval $[0, \frac{\pi}{2}]$ hours. **Use your calculator to evaluate your integral.**
 - Examine your answers for parts (a) and (c). What can you conclude about the motion of the particle in the time interval $[0, \frac{\pi}{2}]$ hours? Explain



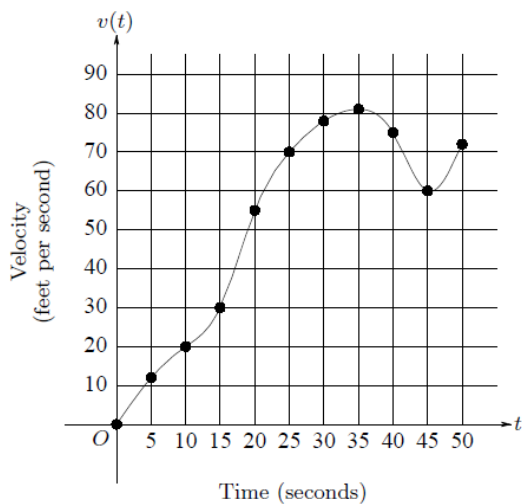
4. The graph of the velocity of a particle moving on the x -axis is given. The particle starts at $x = 2$ when $t = 0$.
- Set up an integral to find the displacement of the particle in the interval $[0, 4]$. **Use the graph to the left to evaluate your integral** and find the displacement.
 - Use your answer to part (a) to find the position of the particle at time $t = 4$.
- c) Set up an integral to find the total distance traveled by the particle in the interval $[0, 4]$. **Use the graph provided to evaluate your integral.**



5. The graph of the velocity of a particle moving on the y -axis is given. The particle starts at $y = 3$ when $t = 0$.
- During what intervals of time is the acceleration of the particle positive? Explain.
 - What is the acceleration of the particle at $t = 4$? Is the particle speeding up or slowing down? Explain.

- Set up an integral to find the displacement of the particle in the interval $[0, 7]$. **Use the graph above to evaluate your integral.** Indicate units of measure.
- Find where the particle is at $t = 7$. (*Hint*: use your answer from part (c)).
- Set up an integral expression to find the average velocity of the particle over the interval $[0, 7]$. **Use the graph above to evaluate your expression.** Indicate units of measure.

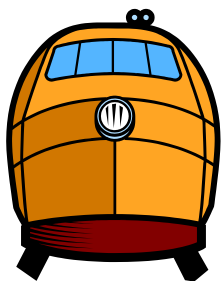
ESSAY FROM 1998 AB EXAM (you may use a graphing calculator for this question):



t (seconds)	$v(t)$ (feet per second)
0	0
5	12
10	20
15	30
20	55
25	70
30	78
35	81
40	75
45	60
50	72

6. The graph of the velocity $v(t)$, in ft/sec, of a car traveling on a straight road, for $0 \leq t \leq 50$, is shown above. A table of values for $v(t)$, at 5 second intervals of time t , is shown to the right of the graph.
- During what intervals of time is the acceleration of the car positive? Give a reason for your answer.
 - Find the average acceleration of the car, in ft/sec^2 , over the interval $0 \leq t \leq 50$.
 - Find one approximation for the acceleration of the car, in ft/sec^2 , at $t = 40$. Show the computations you used to arrive at your answer.
 - Approximate $\int_0^{50} v(t) \cdot dt$ with a Riemann sum, using the midpoints of five subinterval of equal length. Using correct units, explain the meaning of this integral.

-
7. Mary's car runs out of gas as she is driving up a long hill. She lets the car roll without putting on the brakes. As it slows down, stops, and starts rolling backwards, its velocity up the hill is given by $v(t) = 60 - 2t$, where the velocity is measured in ft/sec and t is the number of seconds since the car run out of gas. (**Do not use your calculator** for any of these questions.)
- When does the car stop?
 - What is the car's net displacement between $t = 10$ and $t = 40$ sec?
 - What is the total distance the car rolls between $t = 10$ and $t = 40$ sec?
-



<i>Time (sec)</i>	0	1	2	3	4	5	6	7	8	9	10
<i>Velocity (in/sec)</i>	0	12	22	10	5	13	11	6	2	6	0

8. The table above shows the velocity of a model train engine moving along a track for 10 seconds. Estimate the distance traveled by the train,
- using **right endpoints** of 10 subintervals of equal length
 - using **left endpoints** of 10 subintervals of equal length
 - using the **trapezoidal rule** and 10 subintervals of equal length
 - What's special about all these answers? Why is this happening?
-



More Motion Problems

1. a) Right: $[0, \pi/2)$ and $(3\pi/2, 2\pi]$. Left: $(\pi/2, 3\pi/2)$ (in hours)

b) $s\left(\frac{\pi}{2}\right) - s(0) = \int_0^{\pi/2} (5 \cos t) dt = \sin t \Big|_0^{\pi/2} = 5$ miles

$$s\left(\frac{3\pi}{2}\right) - s\left(\frac{\pi}{2}\right) = \int_{\pi/2}^{3\pi/2} (5 \cos t) dt = \sin t \Big|_{\pi/2}^{3\pi/2} = -10 \text{ miles}$$

$$s(2\pi) - s\left(\frac{3\pi}{2}\right) = \int_{3\pi/2}^{2\pi} (5 \cos t) dt = \sin t \Big|_{3\pi/2}^{2\pi} = 5 \text{ miles}$$

- c) Displacement: 0 miles; total distance: 20 miles

d) $\int_0^{2\pi} (5 \cos t) dt = 0$ miles

e) $\int_0^{2\pi} |5 \cos t| dt = 20$ miles

2. a) $v(t) = \frac{2}{3}t^{3/2} - 18$

b) $s(16) - s(0) = \int_0^{16} v(t) \cdot dt = -14.933$ ft

c) Total distance = $\int_0^{16} |v(t)| \cdot dt = 179.467$ ft

3. a) $s\left(\frac{\pi}{2}\right) - s(0) = 8$ kilometers

b) $v(t) = 8 \cos t$

c) Total distance = $\int_0^{\pi/2} |v(t)| \cdot dt = 8$ kilometers

- d) Since the displacement is positive and it equals the total distance traveled, the particle moved always in the positive direction without ever switching directions.

4. a) $s(4) - s(0) = \int_0^4 v(t) \cdot dt = 1$
 b) $s(4) - s(0) = 1 \Rightarrow s(4) = s(0) + 1 = 3$
 c) Total distance = $\int_0^4 |v(t)| \cdot dt = 5$
5. a) Acceleration is positive when then velocity increases: (0, 2) and (6, 7)
 b) $a(4) = -1 \text{ m/sec}^2$; Since $a(4) < 0$ and $v(4) > 0 \Rightarrow$ Particle is slowing down
 c) $s(7) - s(0) = \int_0^7 v(t) \cdot dt = 3$ meters
 d) $s(7) - s(0) = 3 \Rightarrow s(7) = s(0) + 3 = 6$ meters
 e) average velocity = $\frac{1}{7-0} \int_0^7 v(t) \cdot dt = \frac{s(7) - s(0)}{7-0} = \frac{3}{7} \text{ m/sec}$

6. (a) Acceleration is positive on (0, 35) and (45, 50) because the velocity $v(t)$ is increasing on [0, 35] and [45, 50]

$$3 \begin{cases} 1: (0, 35) \\ 1: (45, 50) \\ 1: \text{reason} \end{cases}$$

Note: ignore inclusion of endpoints

(b) Avg. Acc. = $\frac{v(50) - v(0)}{50 - 0} = \frac{72 - 0}{50} = \frac{72}{50}$
 or 1.44 ft/sec^2

1: answer

- (c) Difference quotient; e.g.

$$\frac{v(45) - v(40)}{5} = \frac{60 - 75}{5} = -3 \text{ ft/sec}^2 \text{ or}$$

$$\frac{v(40) - v(35)}{5} = \frac{75 - 81}{5} = -\frac{6}{5} \text{ ft/sec}^2 \text{ or}$$

$$\frac{v(45) - v(35)}{10} = \frac{60 - 81}{10} = -\frac{21}{10} \text{ ft/sec}^2$$

-or-

Slope of tangent line, e.g.

$$\text{through } (35, 90) \text{ and } (40, 75): \frac{90 - 75}{35 - 40} = -3 \text{ ft/sec}^2$$

$$2 \begin{cases} 1: \text{method} \\ 1: \text{answer} \end{cases}$$

Note: 0/2 if first point not earned

(d) $\int_0^{50} v(t) dt$
 $\approx 10[v(5) + v(15) + v(25) + v(35) + v(45)]$
 $= 10(12 + 30 + 70 + 81 + 60)$
 $= 2530 \text{ feet}$

This integral is the total distance traveled in feet over the time 0 to 50 seconds.

$$3 \begin{cases} 1: \text{midpoint Riemann sum} \\ 1: \text{answer} \\ 1: \text{meaning of integral} \end{cases}$$

7. a) $v = 0 \Rightarrow t = 30 \text{ sec}$

b) $\int_{10}^{40} (60 - 2t) dt = (60t - t^2) \Big|_{10}^{40} = 300 \text{ ft}$

c) $\int_{10}^{40} |60 - 2t| dt = 500 \text{ ft}$ (graph the “V-shape” graph of the absolute value and find areas)

8. a) 87 in

b) 87 in

c) 87 in

d) All 3 approximations yield the same value because the function is equal to zero both at the beginning and end of the interval.