BC Calculus
Section 9.3 – Taylor's Theorem
Lagrange Error Additional Practice

Name: ______________________________

For Problems 1 and 2, use Taylor's Theorem to determine the error bounds of the approximations.

1. $\cos(0.3) \approx 1 - \dfrac{(0.3)^2}{2!} + \dfrac{(0.3)^4}{4!}$

2. $e \approx 1 + 1 + \dfrac{1}{2} + \dfrac{1}{6} + \dfrac{1}{24}$

3. a. Find a 4th degree Taylor polynomial for $\ln x$ centered at $x = 4$.
   b. Find the Lagrange error bound for the polynomial on the interval [4, 4.5].

4. Let $f(x)$ be a function that is continuous and differentiable at all real numbers, and let $f(3) = 1$, $f'(3) = 3$, $f''(3) = 7$, and $f'''(3) = 5$.

   a. Write a 3rd order Taylor polynomial for $f(x)$ about 3.
   b. If $f^{(4)}(x) \le 6$ for all x, find the Lagrange error bound for the polynomial on the interval [2.9, 3.0].

5. Let f be a function having derivatives of all orders for all real numbers. The third-degree Taylor polynomial for f about $x = 2$ is given by $P_3(x) = 7 - 9(x-2)^2 - 3(x-2)^3$.

   Suppose the fourth derivative of f satisfies the inequality $\left|f^{(4)}(x)\right| \le 6$ for all x on the closed interval [0, 2]. Use the Lagrange error bound to justify why $f(0)$ is negative.

6. Use graphs to find a Taylor polynomial $P_n(x)$ for $\cos x$ so that $\left|P_n(x) - \cos x\right| < 0.001$ for every x in $[-\pi, \pi]$.

7. For approximately what values of x can you replace $\sin x$ by $x - \dfrac{x^3}{3!} + \dfrac{x^5}{5!}$ with an error magnitude no greater than 5×10^{-4}?

   a. Find the interval using the Remainder Estimation Theorem.
   b. Find the interval graphically.

Answers:

1. $\left|R_5(0.3)\right| \le 1.013\times 10^{-6}$
2. $\left|R_4(1)\right| \le 0.02266$
3. a. $P_4(x) = \ln 4 + \frac{1}{4}(x-4) - \frac{1}{32}(x-4)^2 + \frac{1}{192}(x-4)^3 - \frac{1}{1024}(x-4)^4$

   b. $\left|R_4(x)\right| \le 6.104\times 10^{-6}$
4. a. $P_3(x) = 1 + 3(x-3) + \frac{7}{2}(x-3)^2 + \frac{5}{6}(x-3)^3$

   b. $\left|R_3(x)\right| \le .000025$
5. $P_3(0) = -5$ and $\left|R_3(x)\right| \le 4$ on [0, 2]; $\therefore -9 \le f(0) \le -1$.
6. $P_{12}(x)$
7. a. $-1.141 \le x \le 1.141$

   b. $-1.144 \le x \le 1.144$