Worksheet: Integration by Trigonometric Substitution

$$a^2 + u^2 \quad (u = a\,tan\theta) \quad ; \quad a^2 - u^2 \quad (u = a\,sin\theta) \quad ; \quad u^2 - a^2 \quad (u = a\,sec\theta)$$

1. Evaluate the following indefinite integrals.

(1) $\displaystyle\int \frac{1}{\sqrt{1 - 4x^2}}\,dx$ 　　　 (2) $\displaystyle\int \frac{1}{x^2 + 25}\,dx$ 　　　 (3) $\displaystyle\int \frac{x}{x^4 + 16}\,dx$

(4) $\displaystyle\int \frac{1}{\sqrt{2 - 5x^2}}\,dx$ 　　　 (5) $\displaystyle\int \frac{3}{x\sqrt{x^2 - 9}}\,dx$ 　　　 (6) $\displaystyle\int \frac{x}{\sqrt{16 - 9x^4}}\,dx$

(7) $\displaystyle\int \frac{1}{x\sqrt{16x^2 - 9}}\,dx$ 　　　 (8) $\displaystyle\int \frac{e^x}{7 + e^{2x}}\,dx$ 　　　 (9) $\displaystyle\int \frac{\sin x}{\sqrt{2 - \cos^2 x}}\,dx$

(10) $\displaystyle\int \frac{1}{\sqrt{x}(1 + x)}\,dx$ 　　　 (11) $\displaystyle\int \frac{1}{x^2\sqrt{4 - x^2}}\,dx$ 　　　 (12) $\displaystyle\int \frac{1}{x\sqrt{x^2 + 4}}\,dx$

(13) $\displaystyle\int \frac{\sqrt{9 - x^2}}{x^2}\,dx$ 　　　 (14) $\displaystyle\int \frac{1}{x\sqrt{25 - x^2}}\,dx$ 　　　 (15) $\displaystyle\int \frac{1}{\sqrt{x^2 - a^2}}\,dx$

(16) $\displaystyle\int \sqrt{1 - x^2}\,dx$ 　　　 (17) $\displaystyle\int \frac{1}{x^2\sqrt{x^2 - 7}}\,dx$ 　　　 (18) $\displaystyle\int x^2\sqrt{16 - x^2}\,dx$

(19) $\displaystyle\int \frac{1}{(x^2 + 4)^{\frac{3}{2}}}\,dx$ 　　　 (20) $\displaystyle\int \frac{1}{(4x^2 - 9)^{\frac{3}{2}}}\,dx$ 　　　 (21) $\displaystyle\int \frac{1}{x^4\sqrt{16 + x^2}}\,dx$

(22) $\displaystyle\int \frac{1}{x\sqrt{x^4 - 4}}\,dx$ 　　　 (23) $\displaystyle\int \frac{\sec^2 x}{(4 - \tan^2 x)^{\frac{3}{2}}}\,dx$ 　　　 (24) $\displaystyle\int \frac{e^{-x}}{(9e^{-2x} + 1)^{\frac{3}{2}}}\,dx$

(25) $\displaystyle\int \frac{1}{\sqrt{x^2 + 2x}}\,dx$ 　　　 (26) $\displaystyle\int \frac{1}{x^4\sqrt{x^2 + 3}}\,dx$ 　　　 (27) $\displaystyle\int \frac{\sqrt{x^2 - 25}}{x}\,dx$

ANSWERS

1. $\dfrac{1}{2}\arcsin 2x + C$

2. $\dfrac{1}{5}\arctan(\dfrac{x}{5}) + C$

3. $\dfrac{1}{8}\arctan(\dfrac{x^2}{4}) + C$

4. $\dfrac{1}{\sqrt{5}}\arcsin(\dfrac{x\sqrt{5}}{\sqrt{2}}) + C$

5. $\sec^{-1}(\dfrac{x}{3}) + C$

6. $\dfrac{1}{6}\arcsin(\dfrac{3x^2}{4}) + C$

7. $\dfrac{1}{3}\sec^{-1}(\dfrac{4x}{3}) + C$

8. $\dfrac{1}{\sqrt{7}}\arctan(\dfrac{e^x}{\sqrt{7}}) + C$

9. $-\arcsin(\dfrac{\cos x}{\sqrt{2}}) + C$

10. $2\arctan\sqrt{x} + C$

11. $-\dfrac{\sqrt{4-x^2}}{4x} + C$

12. $\dfrac{1}{2}\ln\left|\dfrac{\sqrt{x^2+4}-2}{x}\right| + C$

13. $-\dfrac{\sqrt{9-x^2}}{x} - \arcsin(\dfrac{x}{3}) + C$

14. $\dfrac{1}{5}\ln\left|\dfrac{5-\sqrt{25-x^2}}{x}\right| + C$

15. $\ln\left|\dfrac{x+\sqrt{x^2-a^2}}{a}\right| + C$

16. $\dfrac{1}{2}\arcsin x + \dfrac{x\sqrt{1-x^2}}{2} + C$

17. $\dfrac{\sqrt{x^2-7}}{7x} + C$

18. $32\arcsin(\dfrac{x}{4}) - \dfrac{x\sqrt{16-x^2}}{4}(8-x^2) + C$

19. $\dfrac{x}{4\sqrt{x^2+4}} + C$

20. $-\dfrac{x}{9\sqrt{4x^2-9}} + C$

21. $\dfrac{(x^2-8)\sqrt{16+x^2}}{384x^3} + C$

22. $\dfrac{1}{4}\sec^{-1}(\dfrac{x^2}{2}) + C$

23. $\dfrac{\tan x}{4\sqrt{4-\tan^2 x}} + C$

24. $-\dfrac{e^{-x}}{\sqrt{9e^{-2x}+1}} + C$

25. $\ln|x+1+\sqrt{x^2+2x}| + C$

26. $\dfrac{(2x^2-3)\sqrt{x^2+3}}{27x^3} + C$

27. $\sqrt{x^2-25} - 5\sec^{-1}(\dfrac{x}{5}) + C$