

Factoring Cubic Polynomials - Method 1 - Take Out Common Factors

EXAMPLE

$$3x^3 + 12x^2 - 15x = 0$$

$$3x(x^2 + 4x - 5) = 0 \quad \text{Pull out common factor of } 3x$$

$$3x(x - 1)(x + 5) = 0 \quad \text{Factor resulting quadratic expression (find two numbers that multiply to } -5 \text{ and add to } 4)$$

$$x = 0, 1, -5 \quad \text{Find solutions/zeros (values that make each factor equal 0)}$$

Solve each polynomial equation using factoring

1. $x^3 - 9x^2 - 22x = 0$

2. $2x^3 + 6x^2 - 36x = 0$

3. $2x^3 - 7x^2 - 15x = 0$

4. $4x^3 - 18x^2 + 14x = 0$

Factoring Cubic Polynomials - Method 2 - Break Into Pairs and Take Out Common Factors from each Pair

EXAMPLE

$$x^3 + 6x^2 - 4x - 24 = 0$$

$$x^2(x + 6) - 4(x + 6) = 0 \quad \text{Pull out common factor of } x^2 \text{ from first pair and } -4 \text{ from second pair.}$$

$$(x + 6)(x^2 - 4) = 0 \quad \text{Factor common } x + 6 \text{ from both parts of expression}$$

$$(x + 6)(x - 2)(x + 2) = 0 \quad \text{Factor difference of squares}$$

$$x = -6, 2, -2 \quad \text{Find solutions/zeros (values that make each factor equal 0)}$$

Solve each polynomial equation using factoring

5. $x^3 - 7x^2 - 9x + 63 = 0$

6. $x^3 + 10x^2 - 16x - 160 = 0$

7. $2x^3 - 11x^2 - 8x + 44 = 0$

8. $4x^3 + 8x^2 - 9x - 18 = 0$

Solving using a graph, synthetic division, and factoring

If you cannot factor using one of the methods above, you can still solve a polynomial equation using the following method:

1. Graph the polynomial to estimate one zero (one point where the graph crosses the x -axis).
2. Verify that this value is a zero by evaluating the polynomial at this value of x (plugging into the equation).
3. Use this value of x to complete Synthetic Division for the polynomial.
4. Factor the resulting polynomial that you get from Synthetic Division to find the remaining zeroes.

Example

$$x^3 + 5x^2 + 7x + 3 = 0$$

From graph, -1 appears to be a zero.

$$(-1)^3 + 5(-1)^2 + 7(-1) + 3 = 0$$

Check that -1 is a zero.

$$\begin{array}{r|rrrr} -1 & 1 & 5 & 7 & 3 \\ & & -1 & -4 & -3 \\ \hline & 1 & 4 & 3 & 0 \end{array}$$

Synthetic Division

$$x^2 + 4x + 3 = 0$$

Result from Synthetic Division (Quotient)

$$(x + 3)(x + 1) = 0$$

Factors

$$x = -1 \text{ (from first zero and factoring), } -3$$

Solve each polynomial equation using a graph, synthetic division, and factoring

9. $x^3 - 7x + 6 = 0$

10. $x^3 - 4x^2 - 3x + 18 = 0$

11. $-x^3 - 3x^2 + 4x$

12. $x^3 - 3x - 2 = 0$

13. $2x^3 - 6x + 4 = 0$

14. $2x^3 + 10x^2 + 14x + 6 = 0$