

① $y = \frac{-1}{x+1}$ $[1, 3/2]$

$$f(x) = \frac{-1}{x+1} \quad f(3/2) = \frac{-1}{3/2+1} = \frac{-1}{(5/2)} = \frac{-2}{5}$$

$$f(1) = \frac{-1}{1+1} = \frac{-1}{2}$$

$$m = \frac{f(3/2) - f(1)}{3/2 - 1} = \frac{\frac{-2}{5} - \frac{-1}{2}}{\frac{3}{2} - 1} = \frac{\left(\frac{-4}{10} + \frac{5}{10}\right)}{\left(\frac{3}{2} - \frac{2}{2}\right)} = \frac{\left(\frac{+1}{10}\right)}{\left(\frac{1}{2}\right)}$$

$$m = \frac{+1}{10} \cdot \frac{2}{1} = \boxed{\frac{+1}{5}} \quad \textcircled{B}$$

② $y = -2x^2 - 5$ AGAIN, SINCE IT'S multiple choice... you CAN just cheat and derive the FAST WAY... $y' = -4x - 0$

$$\boxed{y' = -4x} \quad \textcircled{C}$$

but... if you insist on doing it the long way... (BAD IDEA)

$$y' = \lim_{h \rightarrow 0} \frac{y(x+h) - y(x)}{h} = \lim_{h \rightarrow 0} \frac{-2(x+h)^2 - 5 - (-2x^2 - 5)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{-2(x^2 + 2xh + h^2) - 5 + 2x^2 + 5}{h}$$

$$= \lim_{h \rightarrow 0} \frac{-2x^2 - 4xh - 2h^2 - 5 + 2x^2 + 5}{h} = \lim_{h \rightarrow 0} \frac{-4xh - 2h^2}{h}$$

$$= \lim_{h \rightarrow 0} \frac{h(-4x - 2h)}{h} = -4x - 0 = \boxed{-4x}$$

③ $y = \frac{1}{x+4} = (x+4)^{-1}$ AGAIN, SINCE IT'S IN
 YOU CAN CHEAT AND
 DERIVE QUICKLY W/ THE
 CHAIN RULE...

$$y' = -1(x+4)^{-2} (1) = \boxed{\frac{-1}{(x+4)^2}} = \frac{-1}{x^2+8x+16} \quad \textcircled{C}$$

$= \boxed{\frac{-1}{x^2+8x+16}}$

④ INSTANTANEOUS rate of
 change is the derivative
 $y = -2x^2 - x - 2$ at $x=0$

$$y' = -4x - 1$$

$$y'(0) = -4(0) - 1 = \boxed{-1} \quad \textcircled{B}$$

⑤ $y = x^2 - 1$ POINT $(0, -1)$
 for the EQUATION of ANY line, you need
 1 A POINT (given... YAY... $(0, -1)$) AND A
 Slope \rightarrow tangent slope is the
derivative evaluated at the point given.

$$y' = 2x$$

$$m_T = 2(0) = 0$$

$$y - (-1) = 0(x - 0)$$

$$y + 1 = 0$$

$$\boxed{y = -1} \quad \textcircled{B}$$

$$\textcircled{6} y = -3x^2 + \frac{1}{3}x^{-3}$$

$$y' = \frac{dy}{dx} = -3(2)x' + \frac{1}{3}(-3)x^{-4} = -6x - x^{-4}$$

$$\boxed{y' = -6x - \frac{1}{x^4}} \quad \textcircled{B}$$

$$\textcircled{7} y = \frac{-5}{4}x - \frac{3}{4}x^{2/5} + x^{-2}$$

$$y' = \frac{-5}{4} - \frac{3}{4}\left(\frac{2}{5}\right)x^{2/5-1} + (-2)x^{-3}$$

$$y' = \frac{-5}{4} - \frac{6}{20}x^{-3/5} - 2x^{-3} = \frac{-5}{4} - \frac{3}{10}x^{-3/5} - 2x^{-3}$$

$$\boxed{y' = \frac{-5}{4} - \frac{3}{10}x^{3/5} - \frac{2}{x^3}} \quad \textcircled{B}$$

$$\textcircled{8} y = x^5 + 2x \quad \text{FIND } \frac{d^3y}{dx^3} = y'''$$

$$y' = 5x^4 + 2$$

$$y'' = 20x^3$$

$$\longrightarrow \boxed{y''' = 60x^2} \quad \textcircled{A}$$

9 $y = (-3x^4 + 5)(x^2 - 2)$ OR FOIL OUT FIRST... UP TO YOU...

u v

$$y' = (-3x^4 + 5)(2x) + (x^2 - 2)(-12x^3)$$

u dv v du

$$y' = -6x^5 + 10x - 12x^5 + 24x^3$$

$$y' = -18x^5 + 24x^3 + 10x \quad \text{(B)}$$

10 $y = \frac{3x^3 - 5}{4x^4 + 2}$ $\leftarrow H$ use the quotient rule...
 $4x^4 + 2 \leftarrow L$

$$y' = \frac{LdH - HdL}{L^2} = \frac{(4x^4 + 2)(9x^2) - (3x^3 - 5)(16x^3)}{(4x^4 + 2)^2}$$

$$= \frac{36x^6 + 18x^2 - 48x^6 + 80x^3}{(4x^4 + 2)(4x^4 + 2)}$$

$$= \frac{-12x^6 + 80x^3 + 18x^2}{16x^8 + 16x^4 + 4} = \frac{2(-6x^6 + 40x^3 + 9x^2)}{2(8x^8 + 8x^4 + 2)}$$

$$= \frac{-6x^6 + 40x^3 + 9x^2}{8x^8 + 8x^4 + 2} \leftarrow \text{still (D)}$$

Answer (D)
 but clean it up if you want

chab - ch.3 review ANS

10) $f(x) = \underbrace{(5x^4 - 3)}_{\text{IN}}^3 \leftarrow \text{out}$ (B)

$$f'(x) = 3(5x^4 - 3)^2 \cdot 20x^3 = \boxed{60x^3(5x^4 - 3)^2}$$

12) $f(x) = \sqrt{\underbrace{(x^2 - 2)^{1/3} - 1}_{\text{IN}}} = \left[\underbrace{(x^2 - 2)^{1/3} - 1}_{\text{IN}} \right]^{1/2} \leftarrow \text{out}$

$$f'(x) = \frac{1}{2} \left[(x^2 - 2)^{1/3} - 1 \right]^{-1/2} \cdot \frac{1}{3} (x^2 - 2)^{-2/3} \cdot 2x \leftarrow \text{ANS (A)}$$

but clean
it up if you
want

$$f'(x) = \frac{2x}{2 \cdot 3 \cdot ((x^2 - 2)^{1/3} - 1)^{1/2} (x^2 - 2)^{2/3}}$$

$$= \frac{x}{3 \cdot ((x^2 - 2)^{1/3} - 1)^{1/2} (x^2 - 2)^{2/3}}$$

← still
(A)

chab - ch-3 review solns

13 given

x	f(x)	f'(x)	g(x)	g'(x)
1	4	-2	3	-2
2	2	$-\frac{3}{2}$	1	$-\frac{1}{2}$
3	1	0	2	$\frac{3}{2}$
4	2	1	4	2

① $h_1(x) = f(x) + g(x)$

$$h_1'(x) = f'(x) + g'(x)$$

$$h_1'(2) = f'(2) + g'(2) \leftarrow \text{look at table}$$

$$= -\frac{3}{2} + -\frac{1}{2} = -\frac{4}{2} = \boxed{-2}$$

② $h_2(x) = f(x) - g(x)$

$$h_2'(x) = f'(x) - g'(x)$$

$$h_2'(4) = f'(4) - g'(4) = 1 - 2 = \boxed{-1}$$

(A)

given

x	f(x)	f'(x)	g(x)	g'(x)
1	3	-1	1	1
2	2	-1	2	$\frac{3}{2}$
3	1	0	4	$\frac{1}{2}$
4	2	1	3	-1

14

14 (cont.) $u \ v$

① $h_1(x) = f(x)g(x)$
 $h_1'(x) = f(x)g'(x) + g(x)f'(x)$
 $u \quad dv \quad + \quad v \quad du$
 $h_1'(1) = f(1)g'(1) + g(1)f'(1)$
 use table ...
 $h_1'(1) = 3(1) + (1)(-1) = 3 - 1 = \boxed{2}$

② $h_2(x) = \frac{f(x)}{g(x)}$
 $\frac{f(x) \leftarrow H}{g(x) \leftarrow L}$
 $h_2'(x) = \frac{g(x)f'(x) - f(x)g'(x)}{(g(x))^2}$
 $\frac{L \quad dH \quad - \quad H \quad dL}{L^2}$
 $h_2'(1) = \frac{g(1)f'(1) - f(1)g'(1)}{(g(1))^2}$ use table..
 $h_2'(1) = \frac{1(-1) - 3(1)}{(1)^2} = \frac{-1-3}{1} = \boxed{-4}$ (A)

15 $f(x) = \sin x^4 = \sin(x^4)$ (IT'S NOT $(\sin x)^4 = \sin^4 x$)
 $f'(x) = \cos(x^4) \cdot 4x^3 = \boxed{4x^3 \cos(x^4)}$ (A)

16 $y = \sec(x^5-5)^3$
 $\begin{matrix} \uparrow \\ \text{OUT} \end{matrix} \quad \begin{matrix} \uparrow \\ \text{IN} \end{matrix} \quad \begin{matrix} \uparrow \\ \text{MIO} \end{matrix}$
 $y' = 15x^4 \sec(x^5-5)^3 \tan(x^5-5) \cdot (x^5-5)^2$
 $y' = \underbrace{\sec(x^5-5)^3}_{\text{DOUR}} \tan(x^5-5)^3 \cdot \underbrace{3(x^5-5)^2}_{\text{DMIO}} \cdot \underbrace{5x^4}_{\text{DIN}}$ (C)

17 $x^3 + 2y^2 = -y^3 + 4$

$$3x^2 + 4y \frac{dy}{dx} = -3y^2 \frac{dy}{dx} + 0$$

$$3x^2 = -3y^2 \frac{dy}{dx} - 4y \frac{dy}{dx}$$

$$3x^2 = (-3y^2 - 4y) \frac{dy}{dx}$$

$$\frac{3x^2}{-3y^2 - 4y} = \frac{dy}{dx} = \frac{3x^2}{-1(3y^2 + 4y)} = \boxed{-\frac{3x^2}{3y^2 + 4y}}$$

(C)

18 $z = 2x^2 + 2x^2y + y^2$

$$0 = 4x + \underbrace{2x^2 \frac{dy}{dx}}_{u \, dv} + \underbrace{y(4x)}_{v \, du} + 2y \frac{dy}{dx}$$

$$-4x - 4xy = 2x^2 \frac{dy}{dx} + 2y \frac{dy}{dx}$$

$$-4x - 4xy = (2x^2 + 2y) \frac{dy}{dx}$$

$$\frac{-4x - 4xy}{2x^2 + 2y} = \frac{dy}{dx} = \frac{-2x - 2xy}{x^2 + y}$$

$$\boxed{\frac{dy}{dx} = \frac{-2x - 2xy}{x^2 + y}}$$

(A)

Free-response

19A $f(x) = -4x^3 + x^2 + 10x - 1$

$$f'(x) = -12x^2 + 2x + 10$$

to write EQUATION OF A line, you need A POINT AND A slope...

POINT

$$x = 2$$

so find the y value by plugging into the original fxn...

$$\begin{aligned} f(2) &= -4(2)^3 + 2^2 + 10(2) - 1 \\ &= -4(8) + 4 + 20 - 1 = -32 + 4 + 20 - 1 \\ &= -28 + 20 - 1 = -8 - 1 = -9 \end{aligned}$$

POINT $(2, -9)$

SLOPE - tangent slope is $f'(x)$ evaluated at the given point

$$\begin{aligned} f'(2) &= -12(2^2) + 2(2) + 10 \\ &= -12(4) + 4 + 10 = -48 + 4 + 10 \\ &= -44 + 10 = \boxed{-34 = m_T} \end{aligned}$$

EQ: $\boxed{y + 9 = -34(x - 2)}$ ← ANSWER

19B SAME POINT (b/c IT'S SAME $f(x)$) at the SAME x)

POINT $(2, -9)$

slope OF NORMAL line is perpendicular to the tangent slope... so flip slope, change sign $m_N = \frac{1}{34}$

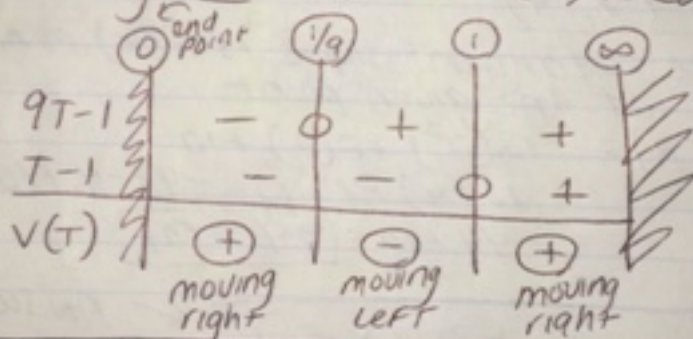
EQ: $\boxed{y + 9 = \frac{1}{34}(x - 2)}$ ← ANSWER

20 $x(t) = 3t^3 - 5t^2 + t + 2 \quad t \geq 0$

A) $x'(t) = v(t) = 9t^2 - 10t + 1$
 $x''(t) = a(t) = 18t - 10$

B) "at rest" means $v=0$
 $0 = 9t^2 - 10t + 1$
 $0 = (9t-1)(t-1)$
 $9t-1=0 \quad t-1=0$
 $t = 1/9 \quad t = 1$

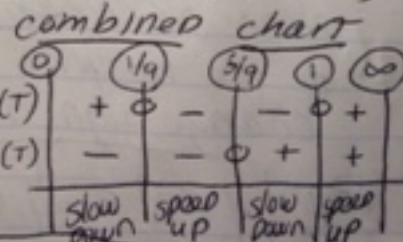
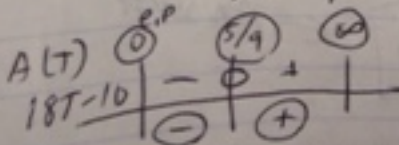
C) make a sign chart for $v(t)$...
 moving left means $v(t)$ is \ominus



particle is moving left $(1/9, 1)$

D) speeding up means $a(t)$ and $v(t)$ have the same sign.
 we already have a $v(t)$ sign chart (part C)
 make $a(t)$ chart:

$a(t) = 18t - 10 = 0$
 $18t = 10$
 $t = 10/18 = 5/9$



SPEED UP: $(1/9, 5/9) \cup (1, \infty)$