1. $y = \frac{-1}{x+1}$ $\quad [1, 3/2]$

$f(x) = \frac{-1}{x+1}$ $\quad f(3/2) = \frac{-1}{\frac{3}{2}+1} = \frac{-1}{\left(\frac{5}{2}\right)} = \frac{-2}{5}$

$f(1) = \frac{-1}{1+1} = \frac{-1}{2}$

$m = \frac{f(3/2) - f(1)}{3/2 - 1} = \frac{-\frac{2}{5} - \frac{-1}{2}}{\frac{3}{2} - 1} = \frac{\left(-\frac{4}{10} + \frac{5}{10}\right)}{\left(\frac{3}{2} - \frac{2}{2}\right)} = \frac{\left(\frac{+1}{10}\right)}{\left(\frac{1}{2}\right)}$

$m = \frac{+1}{10} \cdot \frac{2}{1} = \boxed{\frac{+1}{5}}$ $\quad$ B

2. $y = -2x^2 - 5$ $\quad$ AGAIN, SINCE IT'S multiple choice... you CAN just cheat and derive the FAST WAY... $y' = -4x' - 0$

$\boxed{y' = -4x}$ $\quad$ C

but... IF you insist ON Doing IT the long WAY... (BAD IDEA)

$y' = \lim_{h \to 0} \frac{y(x+h) - y(x)}{h} = \lim_{h \to 0} \frac{-2(x+h)^2 - 5 - (-2x^2 - 5)}{h}$

$= \lim_{h \to 0} \frac{-2(x^2 + 2xh + h^2) - 5 + 2x^2 + 5}{h}$

$= \lim_{h \to 0} \frac{-2x^2 - 4xh - 2h^2 - 5 + 2x^2 + 5}{h} = \lim_{h \to 0} \frac{-4xh - 2h^2}{h}$

$= \lim_{h \to 0} \frac{\cancel{h}(-4x - 2h)}{\cancel{h}} = -4x - \cancel{0} = \boxed{-4x}$

③ $y = \frac{1}{x+4} = (x+4)^{-1}$ AGAIN, since it's m…
you can cheat and Derive quickly w/ the chain rule…

$y' = -1(x+4)^{-2}(1) = \boxed{\frac{-1}{(x+4)^2}} = \frac{-1}{x^2+8x+16}$ Ⓒ

$= \boxed{\frac{-1}{x^2+8x+16}}$

④ INSTANTANEOUS rate of change is the Derivative

$y = -2x^2 - x - 2$ at $x = 0$

$y' = -4x - 1$

$y'(0) = -4(0) - 1 = \boxed{-1}$ Ⓑ

⑤ $y = x^2 - 1$ POINT $(0, -1)$

for the Equation of ANY line, you need 1 A POINT (given… YAY… $(0,-1)$) AND A Slope → tangent slope is the Derivative evaluated At the point given.

$y' = 2x$

$m_T = 2(0) = 0$

$y - (-1) = 0(x - 0)$

$y + 1 = 0$

$\boxed{y = -1}$ Ⓑ

6 $y = -3x^2 + \frac{1}{3}x^{-3}$

$y' = \frac{dy}{dx} = -3(2)x^1 + \frac{1}{3}(-3)x^{-4} = -6x - x^{-4}$

$\boxed{y' = -6x - \frac{1}{x^4}}$ (B)

7 $y = \frac{-5}{4}x - \frac{3}{4}x^{2/5} + x^{-2}$

$y' = \frac{-5}{4} - \frac{3}{4}\left(\frac{2}{5}\right)x^{2/5-1} + (-2)x^{-3}$

$y' = \frac{-5}{4} - \frac{6}{20}x^{-3/5} - 2x^{-3} = \frac{-5}{4} - \frac{3}{10}x^{-3/5} - 2x^{-3}$

$\boxed{y' = \frac{-5}{4} - \frac{3}{10x^{3/5}} - \frac{2}{x^3}}$ (B)

8 $y = x^5 + 2x$ FIND $\frac{d^3y}{dx^3} = y'''$

$y' = 5x^4 + 2$

$y'' = 20x^3 \longrightarrow \boxed{y''' = 60x^2}$ (A)

(9) $y = (-3x^4+5)(x^2-2)$ OR FOIL OUT FIRST... UP TO YOU...

$\quad\;\; u \qquad\quad v$

$y' = (-3x^4+5)(2x) + (x^2-2)(-12x^3)$

$\qquad\; u \qquad\quad dv \qquad\quad v \qquad\quad du$

$y' = -6x^5 + 10x - 12x^5 + 24x^3$

$\boxed{y' = -18x^5 + 24x^3 + 10x}$ (B)

(10) $y = \dfrac{3x^3-5}{4x^4+2} \begin{array}{l}\leftarrow H\\ \leftarrow L\end{array}$ use the quotient rule..

$y' = \dfrac{LdH - HdL}{L^2} = \dfrac{(4x^4+2)(9x^2)-(3x^3-5)(16x^3)}{(4x^4+2)^2}$

↖ Answer (D).. but clean it up if you want

$= \dfrac{36x^6+18x^2-48x^6+80x^3}{(4x^4+2)(4x^4+2)}$

$= \dfrac{-12x^6+80x^3+18x^2}{16x^8+16x^4+4} = \dfrac{\cancel{2}(-6x^6+40x^3+9x^2)}{\cancel{2}(8x^8+8x^4+2)}$

$= \dfrac{-6x^6+40x^3+9x^2}{8x^8+8x^4+2}$ ← still (D) chab - ch.3 review ANS

11 $f(x) = (\underbrace{5x^4-3}_{IN})^3$ ↖ OUT (B)

$f'(x) = 3(5x^4-3)^2 \cdot 20x^3 = \boxed{60x^3(5x^4-3)^2}$

12 $f(x) = \sqrt{(x^2-2)^{1/3}-1} = \Big[\underbrace{(\underbrace{x^2-2}_{IN})^{1/3}-1}_{MID}\Big]^{1/2}$ ↑ OUT

$f'(x) = \frac{1}{2}\left[(x^2-2)^{1/3}-1\right]^{-1/2} \cdot \frac{1}{3}(x^2-2)^{-2/3} \cdot 2x$ ← ANS (A)

but clean it up if you want

$$f'(x) = \frac{\cancel{2}x}{\cancel{2} \cdot 3\left((x^2-2)^{1/3}-1\right)^{1/2}(x^2-2)^{2/3}}$$

$$\boxed{= \frac{x}{3\left((x^2-2)^{1/3}-1\right)^{1/2}(x^2-2)^{2/3}}}$$ ← still (A)

Chab - ch. 3 review solns

(13) given

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	4	-2	3	-2
2	2	$-3/2$	1	$-1/2$
3	1	0	2	$3/2$
4	2	1	4	2

① $h_1(x) = f(x) + g(x)$

$h_1'(x) = f'(x) + g'(x)$

$h_1'(2) = f'(2) + g'(2)$ ← look at table

$= \frac{-3}{2} + \frac{-1}{2} = \frac{-4}{2} = \boxed{-2}$

② $h_2(x) = f(x) - g(x)$

$h_2'(x) = f'(x) - g'(x)$

$h_2'(4) = f'(4) - g'(4) = 1 - 2 = \boxed{-1}$

A

(14) given

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	3	-1	1	1
2	2	-1	2	$3/2$
3	1	0	4	$1/2$
4	2	1	3	-1

14 (cont.)

① $h_1(x) = \overset{u}{f(x)}\,\overset{v}{g(x)}$

$h_1'(x) = \underset{u}{f(x)}\,\underset{dv}{g'(x)} + \underset{v}{g(x)}\,\underset{du}{f'(x)}$

$h_1'(1) = f(1)\,g'(1) + g(1)\,f'(1)$

use table…

$h_1'(1) = 3(1) + (1)(-1) = 3 - 1 = \boxed{2}$

② $h_2(x) = \dfrac{f(x) \leftarrow H}{g(x) \leftarrow L}$

$h_2'(x) = \dfrac{\overset{L}{g(x)}\,\overset{dH}{f'(x)} - \overset{H}{f(x)}\,\overset{dL}{g'(x)}}{\underset{L^2}{(g(x))^2}}$

$h_2'(1) = \dfrac{g(1)\,f'(1) - f(1)\,g'(1)}{(g(1))^2}$ use table…

$h_2'(1) = \dfrac{1(-1) - 3(1)}{(1)^2} = \dfrac{-1-3}{1} = \boxed{-4}$ A

15

$f(x) = \sin x^4 = \underbrace{\sin}_{\text{OUT}}(\underbrace{x^4}_{\text{IN}})$ (IT'S NOT $(\sin x)^4 = \sin^4 x$)

$f'(x) = \cos(x^4) \cdot 4x^3 = \boxed{4x^3\cos(x^4)}$ A

16

$y = \underset{\uparrow \text{ OUT}}{\sec}\,(\underbrace{x^5-5}_{\text{IN}})^{3 \uparrow \text{MID}}$

$\boxed{y' = 15x^4\sec(x^5-5)^3\tan(x^5-5)^3\cdot(x^5-5)^2}$

$y' = \underbrace{\sec(x^5-5)^3\tan(x^5-5)^3}_{d\text{OUT}} \cdot \underbrace{3(x^5-5)^2}_{d\text{MID}} \cdot \underbrace{5x^4}_{D\text{IN}}$ C

(17) $x^3+2y^2=-y^3+4$

$$3x^2+4y\frac{dy}{dx}=-3y^2\frac{dy}{dx}+0$$

$$3x^2=-3y^2\frac{dy}{dx}-4y\frac{dy}{dx}$$

$$3x^2=(-3y^2-4y)\frac{dy}{dx}$$

(C)

$$\frac{3x^2}{-3y^2-4y}=\frac{dy}{dx}=\frac{3x^2}{-1(3y^2+4y)}=\boxed{-\frac{3x^2}{3y^2+4y}}$$

(18) $2=2x^2+\boxed{\underbrace{2x^2}_{u}\underbrace{y}_{v}}+y^2$

$$0=4x+\boxed{\underbrace{2x^2\frac{dy}{dx}}_{u\,Dv}+\underbrace{y(4x)}_{+\,v\,Du}}+2y\frac{dy}{dx}$$

$$-4x-4xy=2x^2\frac{dy}{dx}+2y\frac{dy}{dx}$$

$$-4x-4xy=(2x^2+2y)\frac{dy}{dx}$$

$$\frac{-4x-4xy}{2x^2+2y}=\frac{dy}{dx}=\frac{\cancel{2}(-2x-2xy)}{\cancel{2}(x^2+y)}$$

$$\boxed{\frac{dy}{dx}=\frac{-2x-2xy}{x^2+y}}$$

(A)

Free-response

#19A $f(x) = -4x^3 + x^2 + 10x - 1$

$f'(x) = -12x^2 + 2x + 10$

to write EQUATION OF A line, you need A POINT AND A slope...

POINT

$x = 2$

so find the y value by plugging into the original fxn...

$f(2) = -4(2)^3 + 2^2 + 10(2) - 1$

$= -4(8) + 4 + 20 - 1 = -32 + 4 + 20 - 1$

$= -28 + 20 - 1 = -8 - 1 = -9$

$\boxed{\text{POINT } (2, -9)}$

SLOPE – tangent slope is $f'(x)$ evaluated at the given point

$f'(2) = -12(2^2) + 2(2) + 10$

$= -12(4) + 4 + 10 = -48 + 4 + 10$

$= -44 + 10 = \boxed{-34 = m_T}$

EQ: $\boxed{y + 9 = -34(x - 2)}$ ← ANSWER

#19B SAME POINT (b/c IT'S SAME f(x) at the SAME x)

POINT $(2, -9)$

slope OF NORMAL line is perpendicular to the tangent slope... so FLIP slope, change sign $m_N = \frac{1}{34}$

EQ: $\boxed{y + 9 = \frac{1}{34}(x - 2)}$ ← ANSWER

(20) $x(T) = 3T^3 - 5T^2 + T + 2 \quad T \geq 0$

(A) $\boxed{\begin{array}{l} x'(T) = v(T) = 9T^2 - 10T + 1 \\ x''(T) = A(T) = 18T - 10 \end{array}}$

(B) "AT rest" means $v = 0$

$0 = 9T^2 - 10T + 1$

$0 = (9T - 1)(T - 1)$

$9T - 1 = 0 \quad T - 1 = 0$

$\boxed{T = 1/9 \quad T = 1}$

(C) make a sign chart for $v(T)$...

moving left means $v(T)$ is $-$

	0 (end point)		1/9		1		∞
$9T-1$		$-$	0	$+$		$+$	
$T-1$		$-$		$-$	0	$+$	
$v(T)$		$+$		$-$		$+$	
		moving right		moving left		moving right	

$\boxed{\text{particle is moving left } (1/9, 1)}$

(D) Speeding up means $A(T)$ and $v(T)$ have the same sign.

we already have a $v(T)$ sign chart (part C)

make $A(T)$ chart:

$A(T) = 18T - 10 = 0$

$18T = 10$

$T = 10/18 = \frac{5}{9}$

	0 (e.p)		5/9		∞
$A(T)$ $18T-10$		$-$	0	$+$	
		$-$		$+$	

combined chart

	0		1/9		5/9		1		∞
$v(T)$		$+$	0	$-$		$-$	0	$+$	
$A(T)$		$-$		$-$	0	$+$		$+$	
		slow down		speed up		slow down		speed up	

$\boxed{\text{Speed up: } (1/9, 5/9) \cup (1, \infty)}$