Calculus Honors AB (CHAB) S.Hogan
Chapter 3 Review

This review is identical to the exam in format. Only the actual numerical values in the questions will vary.

Multiple-Choice (#1-18)—See packet. Each problem will be worth 3 points. Only the answer will be graded. (54 points total)

Free-Response: You must SHOW ALL WORK in order to receive full credit.
(22 points total—individual points are listed next to the problems.)

#	PROBLEM
19A (4pts)	Given: $f(x) = -4x^3 + x^2 + 10x - 1$ Find the equation for the TANGENT line at x = 2.
19B (4pts)	Given: $f(x) = -4x^3 + x^2 + 10x - 1$ Find the equation for the **NORMAL** line at x = 2.
20	**For all three parts of question 20, the position of a particle as it moves on the x-axis is given by the function: $x(t) = 3t^3 - 5t^2 + t + 2$ for $t \geq 0$.**
20A (2pts)	Find equations for the velocity, v(t), and acceleration, a(t), of the particle at time t for all $t \geq 0$.
20B (4pts)	At what times is the particle at rest?
20C (4pts)	During what time interval(s) is the particle **moving to the left**?
20C (4pts)	During what time interval(s) is the particle **speeding up**?

CHAB Name_____

Ch.3 Review

For each problem, find the average rate of change of the function over the given interval.

1) $y = -\dfrac{1}{x+1}; \ [1, \dfrac{3}{2}]$

A) 0 B) $\dfrac{1}{5}$

C) $-\dfrac{1}{20}$ D) $-\dfrac{2}{5}$

Use the definition of the derivative to find the derivative of each function with respect to *x*.

2) $y = -2x^2 - 5$

A) $\dfrac{dy}{dx} = -4x^2 + 3$

B) $\dfrac{dy}{dx} = -4x - 9$

C) $\dfrac{dy}{dx} = -4x$

D) $\dfrac{dy}{dx} = -2x - 4$

3) $y = \dfrac{1}{x+4}$

A) $\dfrac{dy}{dx} = -\dfrac{11}{x^2 + 8x + 16}$

B) $\dfrac{dy}{dx} = \dfrac{-x+2}{x^2 + 8x + 16}$

C) $\dfrac{dy}{dx} = -\dfrac{1}{x^2 + 8x + 16}$

D) $\dfrac{dy}{dx} = \dfrac{-x-7}{x^2 + 8x + 16}$

For each problem, find the instantaneous rate of change of the function at the given value.

4) $y = -2x^2 - x - 2; \ 0$

A) 3 B) −1

C) 0 D) −4

For each problem, find the equation of the tangent line to the function at the given point.

5) $y = x^2 - 1; \ (0, -1)$

A) $y = -\dfrac{1}{2}x - 1$

B) $y = -1$

C) $y = -1$

D) $y = -3x - 1$

Differentiate each function with respect to *x*.

6) $y = -3x^2 + \frac{1}{3}x^{-3}$

A) $\frac{dy}{dx} = -3x + \frac{1}{3x^4}$

B) $\frac{dy}{dx} = -6x - \frac{1}{x^4}$

C) $\frac{dy}{dx} = -6x^2 - \frac{1}{x^3}$

D) $\frac{dy}{dx} = -7x$

7) $y = -\frac{5}{4}x - \frac{3}{4}x^{\frac{2}{5}} + x^{-2}$

A) $\frac{dy}{dx} = -\frac{5x}{4} - \frac{3x}{10} - 2x$

B) $\frac{dy}{dx} = -\frac{5}{4} - \frac{3}{10x^{\frac{3}{5}}} - \frac{2}{x^3}$

C) $\frac{dy}{dx} = -\frac{5}{4} - \frac{3}{4x^{\frac{3}{5}}} + \frac{1}{x^3}$

D) $\frac{dy}{dx} = -\frac{5x}{4} - \frac{3x^{\frac{2}{5}}}{10} - \frac{2}{x^2}$

For each problem, find the indicated derivative with respect to *x*.

8) $y = x^5 + 2x$ Find $\frac{d^3y}{dx^3}$

A) $\frac{d^3y}{dx^3} = 60x^2$

B) $\frac{d^3y}{dx^3} = 7x$

C) $\frac{d^3y}{dx^3} = x^2 - 9$

D) $\frac{d^3y}{dx^3} = 125x^5 + 2x$

Differentiate each function with respect to *x*.

9) $y = (-3x^4 + 5)(x^2 - 2)$

A) $\frac{dy}{dx} = (-3x^4 + 5)(x^2 - 2) - 12x^3 \cdot 2x$
$= -3x^6 - 18x^4 + 5x^2 - 10$

B) $\frac{dy}{dx} = (-3x^4 + 5) \cdot 2x + (x^2 - 2) \cdot -12x^3$
$= -18x^5 + 24x^3 + 10x$

C) $\frac{dy}{dx} = (x^2 - 2) \cdot -12x^3$
$= -12x^5 + 24x^3$

D) $\frac{dy}{dx} = (-3x^4 + 5) \cdot 2x$
$= -6x^5 + 10x$

10) $y = \dfrac{3x^3 - 5}{4x^4 + 2}$

A) $\dfrac{dy}{dx} = \dfrac{(4x^4 + 2) \cdot 9x^2 - (3x^3 - 5) \cdot 16x^3}{(3x^3 - 5)^2}$

$= \dfrac{-12x^6 + 80x^3 + 18x^2}{9x^6 - 30x^3 + 25}$

B) $\dfrac{dy}{dx} = \dfrac{9x^2 - 16x^3}{(4x^4 + 2)^2}$

$= \dfrac{-16x^3 + 9x^2}{16x^8 + 16x^4 + 4}$

C) $\dfrac{dy}{dx} = (4x^4 + 2) \cdot 9x^2 - (3x^3 - 5) \cdot 16x^3$

$= -12x^6 + 80x^3 + 18x^2$

D) $\dfrac{dy}{dx} = \dfrac{(4x^4 + 2) \cdot 9x^2 - (3x^3 - 5) \cdot 16x^3}{(4x^4 + 2)^2}$

$= \dfrac{-6x^6 + 40x^3 + 9x^2}{8x^8 + 8x^4 + 2}$

11) $f(x) = (5x^4 - 3)^3$

A) $f'(x) = 3(5x^4 - 3)^2$

B) $f'(x) = 3(5x^4 - 3)^2 \cdot 20x^3$

$= 60x^3(5x^4 - 3)^2$

C) $f'(x) = 20x^3$

D) $f'(x) = (5x^4 - 3)^2 \cdot 20x^3$

12) $f(x) = \sqrt{(x^2-2)^{\frac{1}{3}} - 1}$

A) $f'(x) = \frac{1}{2}\left((x^2-2)^{\frac{1}{3}} - 1\right)^{-\frac{1}{2}} \cdot \frac{1}{3}(x^2-2)^{-\frac{2}{3}} \cdot 2x$

$$= \frac{x}{3\left((x^2-2)^{\frac{1}{3}} - 1\right)^{\frac{1}{2}} \cdot (x^2-2)^{\frac{2}{3}}}$$

B) $f'(x) = \frac{1}{3}(x^2-2)^{-\frac{2}{3}} \cdot 2x$

$$= \frac{2x}{3(x^2-2)^{\frac{2}{3}}}$$

C) $f'(x) = \frac{1}{2}\left((x^2-2)^{\frac{1}{3}} - 1\right)^{-\frac{1}{2}}$

$$= \frac{1}{2\left((x^2-2)^{\frac{1}{3}} - 1\right)^{\frac{1}{2}}}$$

D) $f'(x) = \left((x^2-2)^{\frac{1}{3}} - 1\right)^{-\frac{1}{2}} \cdot \frac{1}{3}(x^2-2)^{-\frac{2}{3}} \cdot 2x$

$$= \frac{2x}{3\left((x^2-2)^{\frac{1}{3}} - 1\right)^{\frac{1}{2}} \cdot (x^2-2)^{\frac{2}{3}}}$$

For each problem, you are given a table containing some values of differentiable functions $f(x)$, $g(x)$ and their derivatives. Use the table data and the rules of differentiation to solve each problem.

13)

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	4	–2	3	–2
2	2	$-\frac{3}{2}$	1	$-\frac{1}{2}$
3	1	0	2	$\frac{3}{2}$
4	2	1	4	2

Part 1) Given $h_1(x) = f(x) + g(x)$, find $h_1'(2)$

Part 2) Given $h_2(x) = f(x) - g(x)$, find $h_2'(4)$

A) $h_1'(2) = -2$
$h_2'(4) = -1$

B) $h_1'(2) = -4$
$h_2'(4) = -1$

C) $h_1'(2) = -3$
$h_2'(4) = -4$

D) $h_1'(2) = -3$
$h_2'(4) = -2$

14)

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	3	−1	1	1
2	2	−1	2	$\frac{3}{2}$
3	1	0	4	$\frac{1}{2}$
4	2	1	3	−1

Part 1) Given $h_1(x) = f(x) \cdot g(x)$, find $h_1'(1)$

Part 2) Given $h_2(x) = \frac{f(x)}{g(x)}$, find $h_2'(1)$

A) $h_1'(1) = 2$
$h_2'(1) = -4$

B) $h_1'(1) = 4$
$h_2'(1) = -1$

C) $h_1'(1) = 5$
$h_2'(1) = -3$

D) $h_1'(1) = -1$
$h_2'(1) = -3$

Differentiate each function with respect to *x*.

15) $f(x) = \sin x^4$

A) $f'(x) = \cos x^4 \cdot 4x^3$
$= 4x^3 \cos x^4$

B) $f'(x) = \tan x^4 \cdot 4x^3$
$= 4x^3 \tan x^4$

C) $f'(x) = -\cos x^4 \cdot 4x^3$
$= -4x^3 \cos x^4$

D) $f'(x) = \sec x^4 \cdot 4x^3$
$= 4x^3 \sec x^4$

16) $y = \sec (x^5 - 5)^3$

A) $\frac{dy}{dx} = -\sec (x^5 - 5)^3 \tan (x^5 - 5)^3 \cdot 3(x^5 - 5)^2 \cdot 5x^4$
$= -15x^4 \sec (x^5 - 5)^3 \cdot \tan (x^5 - 5)^3 \cdot (x^5 - 5)^2$

B) $\frac{dy}{dx} = \sec (x^5 - 5)^3 \cdot \cot (x^5 - 5)^3 \cdot 3(x^5 - 5)^2 \cdot 5x^4$
$= 15x^4 \sec (x^5 - 5)^3 \cdot \cot (x^5 - 5)^3 \cdot (x^5 - 5)^2$

C) $\frac{dy}{dx} = \sec (x^5 - 5)^3 \cdot \tan (x^5 - 5)^3 \cdot 3(x^5 - 5)^2 \cdot 5x^4$
$= 15x^4 \sec (x^5 - 5)^3 \cdot \tan (x^5 - 5)^3 \cdot (x^5 - 5)^2$

D) $\frac{dy}{dx} = \sec (x^5 - 5)^3 \cdot \sec (x^5 - 5)^3 \cdot 3(x^5 - 5)^2 \cdot 5x^4$
$= 15x^4 \cdot \sec^2 (x^5 - 5)^3 \cdot (x^5 - 5)^2$

For each problem, use implicit differentiation to find $\frac{dy}{dx}$ in terms of x and y.

17) $x^3 + 2y^2 = -y^3 + 4$

A) $\frac{dy}{dx} = \frac{-4y - 3y^2}{3x^2}$

B) $\frac{dy}{dx} = -\frac{3x^2}{-y^3 + 4}$

C) $\frac{dy}{dx} = -\frac{3x^2}{4y + 3y^2}$

D) $\frac{dy}{dx} = \frac{x^3 + 2y^2}{-y^3 + 4}$

18) $2 = 2x^2 + 2x^2y + y^2$

A) $\frac{dy}{dx} = \frac{-2x - 2xy}{x^2 + y}$

B) $\frac{dy}{dx} = \frac{4x + 4xy}{2x^2 + 2x^2y + y^2}$

C) $\frac{dy}{dx} = \frac{-x^2 - y}{2x + 2xy}$

D) $\frac{dy}{dx} = \frac{2}{2x^2 + 2x^2y + y^2}$