APBC S.Hogan
Unit 6 Review

The topics on this review are similar to those found on the unit exam. Keep in mind that although the concepts will be the same, you may be required to apply them differently. In addition to this review, you should make sure that you are familiar with all material and formulae from this chapter.

Exam Format:

- 9 multiple-choice questions and 2 free-response—NON-CALCULATOR
- You will NOT be penalized for wrong answers on the multiple-choice, so you should answer EVERY question.

Time Restrictions:

You will have 48 minutes to complete this examination. This means that you are being required to move at AP pace (2 minutes per non-calculator multiple-choice and 15 minutes for free-response.)

PLEASE COMPLETE THIS REVIEW SHEET IN ITS ENTIRETY BEFORE WE DO OUR IN-CLASS REVIEW. THAT IS THE BEST WAY TO PREPARE FOR THIS EXAM.

1 Which of the following series are convergent?

I. $1+\frac{1}{2^2}+\frac{1}{3^2}+\ldots+\frac{1}{n^2}+\ldots$

II. $1+\frac{1}{2}+\frac{1}{3}+\ldots+\frac{1}{n}+\ldots$

III. $1-\frac{1}{3}+\frac{1}{3^2}-\ldots+\frac{(-1)^{n+1}}{3^{n-1}}+\ldots$

(A) I only (B) III only (C) I and III only (D) II and III only (E) I, II, and III

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A series expansion of $\frac{\sin t}{t}$ is

(A) $1-\frac{t^2}{3!}+\frac{t^4}{5!}-\frac{t^6}{7!}+\cdots$

(B) $\frac{1}{t}-\frac{t}{2!}+\frac{t^3}{4!}-\frac{t^5}{6!}+\cdots$

(C) $1+\frac{t^2}{3!}+\frac{t^4}{5!}+\frac{t^6}{7!}+\cdots$

(D) $\frac{1}{t}+\frac{t}{2!}+\frac{t^3}{4!}+\frac{t^5}{6!}+\cdots$

(E) $t-\frac{t^3}{3!}+\frac{t^5}{5!}-\frac{t^7}{7!}+\cdots$

3

The complete interval of convergence of the series $\sum_{k=1}^{\infty}\frac{(x+1)^k}{k^2}$ is

(A) $0<x<2$ (B) $0\le x\le 2$ (C) $-2<x\le 0$

(D) $-2\le x<0$ (E) $-2\le x\le 0$

4

Let f be the function given by $f(x)=\ln(3-x)$. The third-degree Taylor polynomial for f about $x=2$ is

(A) $-(x-2)+\frac{(x-2)^2}{2}-\frac{(x-2)^3}{3}$

(B) $-(x-2)-\frac{(x-2)^2}{2}-\frac{(x-2)^3}{3}$

(C) $(x-2)+(x-2)^2+(x-2)^3$

(D) $(x-2)+\frac{(x-2)^2}{2}+\frac{(x-2)^3}{3}$

(E) $(x-2)-\frac{(x-2)^2}{2}+\frac{(x-2)^3}{3}$

5

The coefficient of x^3 in the Taylor series for e^{3x} about $x=0$ is

(A) $\frac{1}{6}$ (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) $\frac{3}{2}$ (E) $\frac{9}{2}$

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If $\sum_{n=0}^{\infty} a_n x^n$ is a Taylor series that converges to $f(x)$ for all real x, then $f'(1) =$

(A) 0 (B) a_1 (C) $\sum_{n=0}^{\infty} a_n$ (D) $\sum_{n=1}^{\infty} na_n$ (E) $\sum_{n=1}^{\infty} na_n{}^{n-1}$

7

Which of the following series diverge?

I. $\sum_{k=3}^{\infty} \frac{2}{k^2+1}$

II. $\sum_{k=1}^{\infty} \left(\frac{6}{7}\right)^k$

III. $\sum_{k=2}^{\infty} \frac{(-1)^k}{k}$

(A) None (B) II only (C) III only (D) I and III (E) II and III

8

The interval of convergence of $\sum_{n=0}^{\infty} \frac{(x-1)^n}{3^n}$ is

(A) $-3 < x \leq 3$ (B) $-3 \leq x \leq 3$ (C) $-2 < x < 4$

(D) $-2 \leq x < 4$ (E) $0 \leq x \leq 2$

9

If $s_n = \left(\frac{(5+n)^{100}}{5^{n+1}}\right)\left(\frac{5^n}{(4+n)^{100}}\right)$, to what number does the sequence $\{s_n\}$ converge?

(A) $\frac{1}{5}$ (B) 1 (C) $\frac{5}{4}$ (D) $\left(\frac{5}{4}\right)^{100}$ (E) The sequence does not converge.

10 FR	A function f has derivatives of all orders at $x = 0$. Let $P_n(x)$ denote the nth-degree Taylor polynomial for f about $x = 0$. (a) It is known that $f(0) = -4$ and that $P_1\left(\frac{1}{2}\right) = -3$. Show that $f'(0) = 2$. (b) It is known that $f''(0) = -\frac{2}{3}$ and $f'''(0) = \frac{1}{3}$. Find $P_3(x)$. (c) The function h has first derivative given by $h'(x) = f(2x)$. It is known that $h(0) = 7$. Find the third-degree Taylor polynomial for h about $x = 0$.
11 FR	Let $f(x) = \ln\left(1 + x^3\right)$. (a) The Maclaurin series for $\ln(1 + x)$ is $x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots + (-1)^{n+1} \cdot \frac{x^n}{n} + \cdots$. Use the series to write the first four nonzero terms and the general term of the Maclaurin series for f. (b) The radius of convergence of the Maclaurin series for f is 1. Determine the interval of convergence. Show the work that leads to your answer. (c) Write the first four nonzero terms of the Maclaurin series for $f'\left(t^2\right)$. If $g(x) = \int_0^x f'\left(t^2\right)\,dt$, use the first two nonzero terms of the Maclaurin series for g to approximate $g(1)$. (d) The Maclaurin series for g, evaluated at $x = 1$, is a convergent alternating series with individual terms that decrease in absolute value to 0. Show that your approximation in part (c) must differ from $g(1)$ by less than $\frac{1}{5}$.