APBC
Unit 4 Review

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The topics on this review are similar to those found on the unit exam. Keep in mind that although the concepts will be the same, you may be required to apply them differently. In addition to this review, you should make sure that you are familiar with all material and formulae from this chapter.

Exam Format:

(1) 9 multiple-choice questions
(2) 2 free-response question
(3) NO CALCULATORS
(4) You will NOT be penalized for wrong answers on the multiple-choice, so you should answer EVERY question.

Time Restrictions:
You will have 48 minutes to complete this examination. This means that you are being required to move at AP pace (2 minutes per non-calculator multiple-choice and 15 minutes for free-response.)

PLEASE COMPLETE THIS REVIEW SHEET IN ITS ENTIRETY BEFORE WE DO OUR IN-CLASS REVIEW. THAT IS THE BEST WAY TO PREPARE FOR THIS EXAM.

1	If $\frac{dy}{dx} = \sin x \cos^2 x$ and if $y = 0$ when $x = \frac{\pi}{2}$, what is the value of y when $x = 0$? (A) -1 (B) $-\frac{1}{3}$ (C) 0 (D) $\frac{1}{3}$ (E) 1
2	The population $P(t)$ of a species satisfies the logistic differential equation $\frac{dP}{dt} = P\left(2 - \frac{P}{5000}\right)$, where the initial population $P(0) = 3{,}000$ and t is the time in years. What is $\lim_{t\to\infty} P(t)$? (A) $2{,}500$ (B) $3{,}000$ (C) $4{,}200$ (D) $5{,}000$ (E) $10{,}000$
3	If $\frac{dy}{dx} = x^2 y$, then y could be (A) $3\ln\left(\frac{x}{3}\right)$ (B) $e^{\frac{x^3}{3}} + 7$ (C) $2e^{\frac{x^3}{3}}$ (D) $3e^{2x}$ (E) $\frac{x^3}{3} + 1$
4	If $\frac{dy}{dt} = -2y$ and if $y = 1$ when $t = 0$, what is the value of t for which $y = \frac{1}{2}$? (A) $-\frac{\ln 2}{2}$ (B) $-\frac{1}{4}$ (C) $\frac{\ln 2}{2}$ (D) $\frac{\sqrt{2}}{2}$ (E) $\ln 2$

5 NOTE: This problem is from the 1993 Exam, when scientific (non-graphing) calculators were permitted….so you will need to use a calculator to get the final answer, but should be able to do the rest of the steps by hand. No problems on the exam will require a calculator.

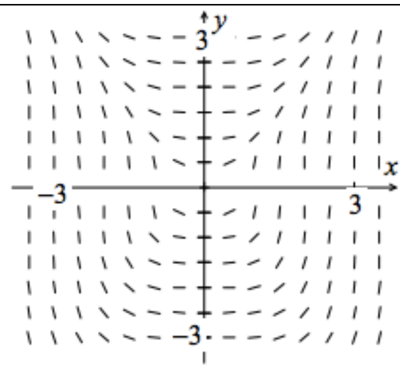
During a certain epidemic, the number of people that are infected at any time increases at a rate proportional to the number of people that are infected at that time. If 1,000 people are infected when the epidemic is first discovered, and 1,200 are infected 7 days later, how many people are infected 12 days after the epidemic is first discovered?

(A) 343 (B) 1,343 (C) 1,367 (D) 1,400 (E) 2,057

6 When $x = 8$, the rate at which $\sqrt[3]{x}$ is increasing is $\frac{1}{k}$ times the rate at which x is increasing. What is the value of k?

(A) 3 (B) 4 (C) 6 (D) 8 (E) 12

7



) Shown above is a slope field for which of the following differential equations?

(A) $\frac{dy}{dx} = \frac{x}{y}$

(B) $\frac{dy}{dx} = \frac{x^2}{y^2}$

(C) $\frac{dy}{dx} = \frac{x^3}{y}$

(D) $\frac{dy}{dx} = \frac{x^2}{y}$

(E) $\frac{dy}{dx} = \frac{x^3}{y^2}$

8 For what value of k will $x + \frac{k}{x}$ have a relative maximum at $x = -2$?

(A) -4 (B) -2 (C) 2 (D) 4 (E) None of these

9 If $h(x) = f^2(x) - g^2(x)$, $f'(x) = -g(x)$, and $g'(x) = f(x)$, then $h'(x) =$

(A) 0 (B) 1 (C) $-4f(x)g(x)$

(D) $(-g(x))^2 - (f(x))^2$ (E) $-2(-g(x) + f(x))$

10 FR

A population is modeled by a function P that satisfies the logistic differential equation

$$\frac{dP}{dt} = \frac{P}{5}\left(1 - \frac{P}{12}\right).$$

(a) If $P(0) = 3$, what is $\lim_{t\to\infty} P(t)$?

If $P(0) = 20$, what is $\lim_{t\to\infty} P(t)$?

(b) If $P(0) = 3$, for what value of P is the population growing the fastest?

(c) A different population is modeled by a function Y that satisfies the separable differential equation

$$\frac{dY}{dt} = \frac{Y}{5}\left(1 - \frac{t}{12}\right).$$

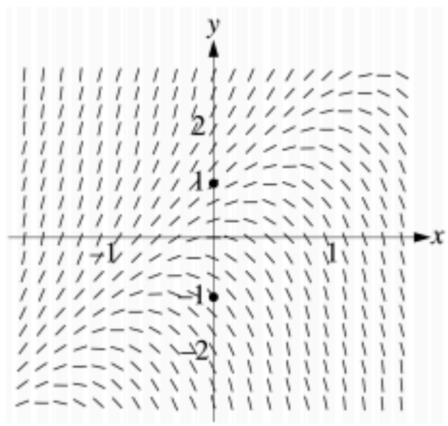
Find $Y(t)$ if $Y(0) = 3$.

(d) For the function Y found in part (c), what is $\lim_{t\to\infty} Y(t)$?

11 FR

Consider the differential equation $\frac{dy}{dx} = 2y - 4x$.

(a) The slope field for the given differential equation is provided. Sketch the solution curve that passes through the point $(0, 1)$ and sketch the solution curve that passes through the point $(0, -1)$.
(Note: Use the slope field provided in the pink test booklet.)



(b) Let f be the function that satisfies the given differential equation with the initial condition $f(0) = 1$. Use Euler's method, starting at $x = 0$ with a step size of 0.1, to approximate $f(0.2)$. Show the work that leads to your answer.

(c) Find the value of b for which $y = 2x + b$ is a solution to the given differential equation. Justify your answer.

(d) Let g be the function that satisfies the given differential equation with the initial condition $g(0) = 0$. Does the graph of g have a local extremum at the point $(0, 0)$? If so, is the point a local maximum or a local minimum? Justify your answer.