APAB and APBC S.Hogan
Chapter Homework Assignment
Name: _____

APAB/BC Competition Review HW

Due on _____ (day AFTER competition) at the end of class.

Directions: Use this sheet as a cover sheet for your homework assignment. If you do not staple this cover sheet to the front of your assignment, you will receive a zero. You may submit your HW early but NO LATE HW will be accepted.

Assignment:

2018 Calculus Competition
2019 Calculus Competition
(Complete ALL problems)

REMEMBER, THIS CALCULUS COMPETITION WILL COUNT AS AN EXAM FOR YOU. YOUR TEAM SCORE WILL BE ADDED TO YOUR INDIVIDUAL SCORE.

Grading:

Item	Possible Points	Score
All Problems Completed	2	
Problems Labeled and in Numerical Order, Page Numbers are Labeled	2	
First Random Problem (correct with sufficient work)	4	
Second Random Problem (correct with sufficient work)	4	
Third Random Problem (correct with sufficient work)	4	
Fourth Random Problem (correct with sufficient work)	4	
TOTAL	20	

Calculus Competition March 16, 2018

Exam Format and Timeline

SET-UP:

Scantrons, Projector, Screen, Dongle, Laptop, Scrap Paper, Buzzers, Extension cords?? PRINT BLANK CERTS!!! AWARDS FROM SPIKES!!

TIME	*LOCATION*	*DETAILS*
9:00AM	*Auditorium*	*Teams Arrive at HSES (Teams will gather in the auditorium)*
9:15-9:35AM	*Auditorium*	*Individual Exam (This will be in the auditorium this year)—Students will be given (a) individual exam papers (facedown) (b) scantron (c) scrap paper. 8 MC, Non-Calculator (20 minutes) (Max possible score is 24 points)*
9:35-9:50AM	*Break*	*Brief Break for scoring/Get ready for Team Round (15 minutes)*
9:50-11:10AM	*Auditorium*	*Team Round—18 Questions. Most are worth 1 point each. A few are worth 2 or 3 points. Maximum possible scores is 24 points. Non-Calculator. (90 minutes)*
11:10-11:25AM	*Auditorium*	*Brief Break for scoring (students will stay in Auditorium) (15 minutes)*
11:25-11:50AM	*Auditorium*	*Final Showdown (with buzzers)--Top three teams (scoring is explained below) will face off in front of all the other teams to determine the 1st, 2nd, and 3rd place.*
NOON	*Auditorium*	*Pizza party in the cafeteria for all participants--If students want pizza/beverage, please have them bring $5...we'll order loads of pizza and the kids can hang out and eat before you all leave. If you have a student who wants pizza but can't swing the $5, let me know and we'll work something out.*

At the conclusion of the final showdown round, all team and individual winners will be recognized. Awards will be presented.

Individual Round: Students will be given an individual exam with 8 multiple-choice problems. Remember, the questions will cover limits, continuity, derivatives, application of derivatives, and basic integration.

Team Round: For each question, the groups will be given a sheet of paper with the free-response question at the top and will be expected to write their answer on the same sheet of paper. The questions will take 3-4 minutes each. The papers will be collected from the teams and checked immediately. After each question, the answer will be reviewed briefly to help students prepare for the AP Exam. I will attempt to post team scores after each question so that the participants have constant updates on the standings.

SCORING: To determine the top three teams for the "final showdown", we will sum the team round score with the top three individual scores on the team (this way we are not penalizing teams with only 3 students.) The three teams with the highest scores will be in the showdown. IF there is a tie that yields more than 3 teams for the showdown, the tie will be broken as follows: only the top scorer on the individual exams will be counted...if there is still a tie, then only the top one scorers on each team will be counted...if there is still a tie, only the top scorers on each team will be counted...if there is still a tie, more than 3 teams will participate in the showdown.

Each team will consist of 3 to 5 students. Schools may bring multiple teams.

Name:	*School:*	*Team#:*

Individual Round: (Non-Calculator)

1

Given $\frac{dy}{dx} = x^2(y+1)$ *and* $y(0) = 5$, *find y(-1)*

A. $4e^{-1/3} - 1$

B. $5e^{-1/3}$

C. $\frac{6-\sqrt[3]{e}}{\sqrt[3]{e}}$

D. $6\sqrt[3]{e} + 1$

E. $6\sqrt[3]{e} - 1$

2

Information about two continuous, differentiable functions, f(x) and g(x), are given in the table below. Given that h(x) = f(x)*g(x), what is the value of h'(2)?

x	f(x)	f '(x)	g(x)	g '(x)
1	2	6	1	0
2	3	-5	-2	1
3	-1	2	4	10

A. -6

B. 13

C. -17

D. -11

E. 6

3

Hawkins Lab is leaking radioactive ooze at a rate of R(t), in gallons per minute. Select values for R(t) are provided in the table below. Using a lower sum as an estimate, what is the least amount of ooze that has leaked out of the lab during the 8-hour period?

t (minutes)	0	1	2	4	5	8
R(t) (gallons/minute)	3	2	7	10	5	2

A. 18 gallons
B. 24 gallons
C. 27 gallons
D. 29 gallons
E. 44 gallons

4	Bailey, the amazing super dog, is running along the x-axis. His position, B(t), can be modeled by the following equation for all t $\geq$ 0. $$B(t) = 2t^3 - \frac{11}{2}t^2 + 3t$$ For what time period(s) is Bailey speeding up? A. $\left(\frac{1}{3}, \frac{11}{12}\right) \cup (\frac{3}{2}, \infty)$ B. $(-\infty, \frac{1}{3}) \cup (\frac{3}{2}, \infty)$ C. $\left[0, \frac{1}{3}\right) \cup (\frac{3}{2}, \infty)$ D. $(-\infty, \frac{11}{12})$ E. $[0, \frac{11}{12})$

5	$F(x) = \int_{x^3}^{5} \cos(t)\, dt$, find F'(x). A. $-\sin(3x^2)$ B. $-\sin(5) + \sin(x^3)$ C. $\cos(5) - \cos(x^3)$ D. $3x^2\cos(x^3)$ E. $-3x^2\cos(x^3)$
6	$f(x) = e^x\cos(2x)$ which of the following accurate describes the function at x = 0? A. positive, decreasing, concave up B. negative, decreasing, concave up C. positive, increasing, concave up D. negative, increasing, concave down E. positive, increasing, concave down

7	$$If\ \frac{d}{dx}f(x) = g(6x)\ \ and \frac{d}{dx}g(x) = f(x/3), then\ \frac{d^2}{dx^2}f(x^2) =$$ A. $6f(2x)$ B. $f(2x^2)$ C. $g(6x^2) + 2xf(2x^2)$ D. $2g(6x^2) + 24x^2f(2x^2)$ E. $12xf(2x^2)$
8	$$\int x^2\sqrt{x+1}\,dx$$ A. $\frac{2}{5}(x+1)^{\frac{5}{2}} - \frac{4}{3}(x+1)^{\frac{3}{2}} + 2(x+1)^{\frac{1}{2}} + C$ B. $\frac{2}{7}(x+1)^{\frac{7}{2}} - \frac{4}{5}(x+1)^{\frac{5}{2}} + \frac{2}{3}(x+1)^{\frac{3}{2}} + C$ C. $\frac{2}{5}(x+1)^{\frac{5}{2}} - \frac{2}{3}(x+1)^{\frac{3}{2}} + C$ D. $\frac{2}{5}x^{\frac{5}{2}} + \frac{x^3}{3} + C$ E. $\frac{2}{9}x^3(x+1)^{\frac{3}{2}} + C$

SCHOOL:	***TEAM #:***

<u>Question 1: (3 minutes)</u>

$$\int \frac{5x^2}{\sin^2(x^3)}\,dx$$

SCHOOL:	***TEAM #:***

<u>Question 2: (3 minutes)—Worth 2 points</u>

A particle moves along the x-axis so that its velocity at time t (where t > 0) is given by

$v(t) = 4tsin(t^2)$.

1. Write an expression for the acceleration, a(t), of the particle at time t.

2. Given that the particle's initial position was at x = 5, write an expression for x(t), the position of the particle at time t.

SCHOOL:	***TEAM #:***

<u>Question 3: (3 minutes)</u>

Given $f(x) = \sin^3(3x)$, *find* $f'\left(\frac{\pi}{9}\right)$.

Fully simplify your answer.

SCHOOL:	***TEAM #:***

Question 4: (3 minutes)—Worth 2 points

The function, G(x), the first derivative, G'(x), and the second derivative, G"(x), are continuous. G(x) has exactly three zeros. G'(x) and G"(x) each have exactly two zeros . Selected values of the function and both derivatives are given in the table below.

x	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10
G(x)	3	2	0	1	5	2	0	-3	-4	-5	-2	-3	0	1	4	8
G'(x)	6	5	3	2	0	-1	-3	-2	-2	-1	0	-1	-3	-4	-5	-3
G"(x)	-4	-2	-1	0	1	2	4	6	7	5	3	2	3	2	0	-1

1. What are the coordinates of the relative maximum of G(x)?

2. If the domain of G(x) is all real numbers on what interval(s) is G(x) positive, decreasing, and concave up?

SCHOOL:	***TEAM #:***

<u>Question 5: (3 minutes)</u>

What are the ***<u>coordinates</u>*** *of the point in the 2nd quadrant on the curve* $y = x^2 + 1$ *that is closest to (0, 3)?*

SCHOOL:	***TEAM #:***

<u>Question 6: (3 minutes)</u>

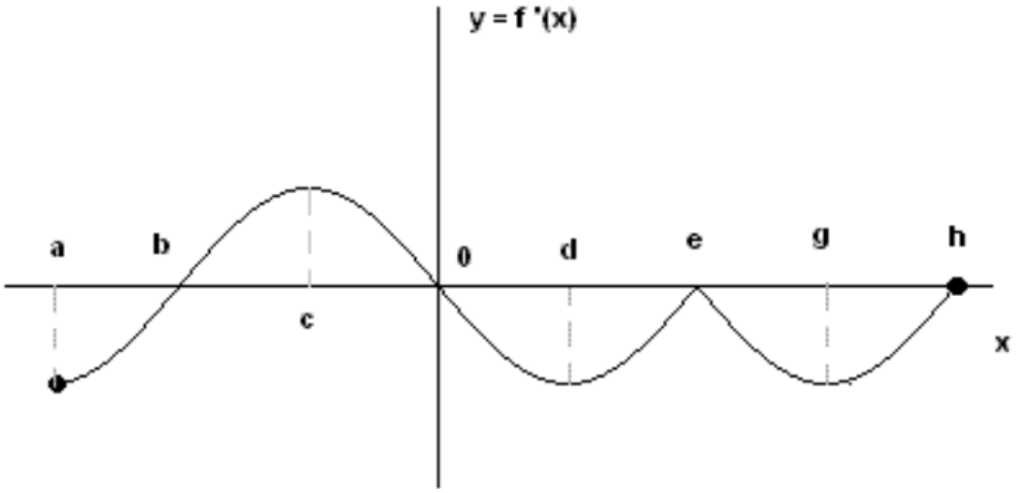
What is the equation of the line **<u>normal</u>** to the curve below at (1, -1)?

$$2x^3 - 5xy^2 = xy - 2$$

SCHOOL:	***TEAM #:***

<u>Question 7</u>: (3 minutes)—Worth 3 points

Below is the graph of f '(x) on the interval [a, h.] f'(x) = 0 at x = b, 0, e, and h. f '(x) is level and has relative extrema at x = a, c, e, and g.



1. On what interval(s) is f(x) decreasing and concave up?

2. For what x value does f(x) have an absolute maximum?

3. For what x value(s) does f(x have a relative minimum?

SCHOOL:	***TEAM #:***

<u>Question 8</u>: (3 minutes)—Worth 2 points

$$f(x) = e^{3x}x^2$$

1. On what intervals of x is f increasing?

*2. What are the **<u>coordinates</u>** of the relative maximum of the graph?*

SCHOOL:	***TEAM #:***

<u>Question 9</u>: (3 minutes)

$$f(x) = \begin{cases} 3ax, & x < 2 \\ ax^3 + bx + 4, & x \geq 2 \end{cases}$$

If f(x) is continuous and differentiable for all real numbers, x, what are the values of a and b?

SCHOOL:	***TEAM #:***

<u>Question 10:</u> ***(3 minutes)***

$$f(x) = \frac{100}{x} + 4$$

Find all values, c, in [1, 5] guaranteed by the Mean Value Theorem for derivatives.

SCHOOL:	***TEAM #:***

<u>Question 11: (3 minutes)</u>

Given: $f(x) = 6x^2 - 8x + 2$

Find the average value of f(x) on [1, 5.]

SCHOOL:	***TEAM #:***

<u>Question 12: (3 minutes)</u>

Evaluate the integral. Fully simplify your answer.

$$\int_0^6 \sqrt{36 - x^2} \; + 4x \, dx =$$

SCHOOL:	***TEAM #:***

<u>Question 13: (3 minutes)</u>

Evaluate the integral. Fully simplify your answer.

$$\int_{0}^{\frac{\pi}{6}} cos^2(2x)\, sin(2x)\, dx$$

SCHOOL:	***TEAM #:***

<u>Question 14: (3 minutes)</u>

For what x value(s) does f(x) have a horizontal tangent line in $[0,2\pi)$?

$$f(x) = \sin^2 x + \sqrt{3}cosx$$

SCHOOL:	***TEAM #:***

<u>Question 15:</u> ***(3 minutes)***

What is the minimum value of f(x) on [-10, 10]?

$$f(x) = x^3 - 12x + 5$$

SCHOOL:	***TEAM #:***

<u>Question 16</u>: (3 minutes)

Evaluate the limit.

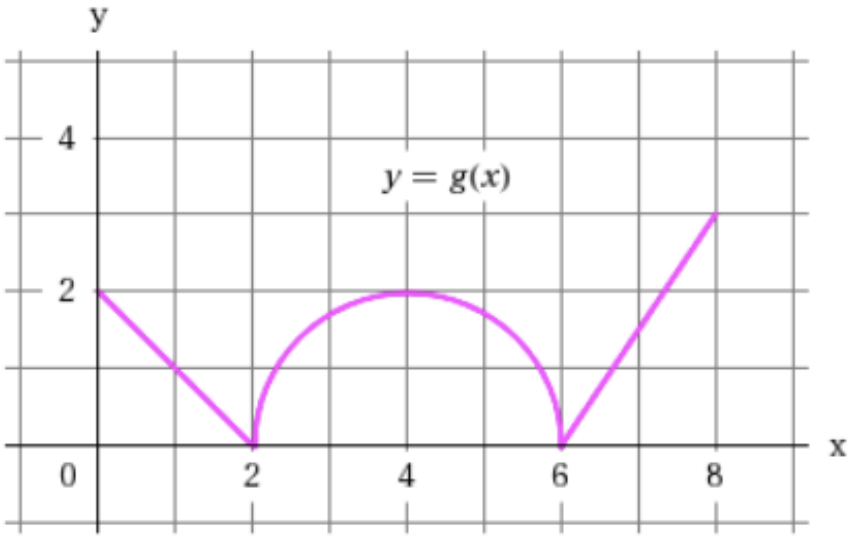
$$\lim_{k \to 0} \frac{\frac{5}{(2+k)^3} - \frac{5}{8}}{k}$$

SCHOOL:	***TEAM #:***

<u>Question 17: (3 minutes)—Worth 2 points</u>

The graph of g(x), shown below, consists of a line segment from x = 0 to x = 2, a semicircle from from x = 2 to x = 6, and a line segment from x = 6 to x = 8.

All answers must be fully simplified to receive credit.



1. *What is the value of* $\int_0^8 g(x)dx$*?*

2. *What is the value of* $\int_2^8 (g(x) + 2)\, dx$*?*

SCHOOL:	***TEAM #:***

Question 18:

Evaluate the limit.

$$\lim_{x \to 0} [1 + sin\ (x)]^{1/x}$$

FINAL SHOWDOWN QUESTION 1:

$$g(x) = \frac{\sqrt{3\pi - \cos(4\pi)}}{e^2 + 4\pi^5} + 3\pi x^2$$

Find g'(x).

FINAL SHOWDOWN QUESTION 2:

Evaluate the limit:

$$\lim_{x \to 3^-} \frac{|3 - x|}{8x - 24}$$

FINAL SHOWDOWN QUESTION 3:

Give the equation of the line normal to the curve $-xy^2 + x^2y + 2 = 0$ *at x = -2.*

FINAL SHOWDOWN QUESTION 4:

Evaluate the limit.

$$\lim_{h \to 0} \frac{\frac{1}{\tan(\theta + h)} - \frac{1}{\tan(\theta)}}{h}$$

FINAL SHOWDOWN QUESTION 5:

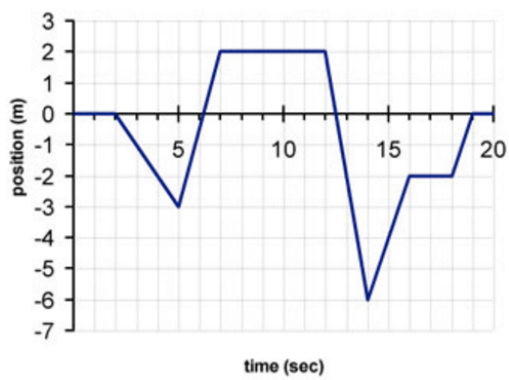
$$f(x) = e^x \cos x \qquad [0, 2\pi)$$

For what value(s) of x does f (x) have a horizontal tangent line?

TIE-BREAK QUESTION:

The position of a particle (in meters) with respect to time (in seconds) is given by the graph below.

Assume all changes in the position graph occur at points with integer coordinates (in case it's difficult to see the grid.)



With correct units, what is the value of v(3)?

Individual Exam (Answers)—NO CALCULATOR		
1	*C*	
2	*B*	
3	*D*	
4	*A*	
5	*E*	
6	*E*	
7	*D*	
8	*B*	
Team Round (Answers)—NO CALCULATOR		
1	$-\frac{5}{3}\cot(x^3)+C$	*1 point*
2	1. $A(t)=8t^2\cos(t^2)+4\sin(t^2)$ 2. $x(t)=-2\cos(t^2)+7$	*2 points*
3	*27/8*	*1 point*
4	1. *(-1, 5)* 2. *(-1,1)U(7,9)*	*2 points*
5	$\left(-\sqrt{\frac{3}{2}},\frac{5}{2}\right)$	*1 points*
6	$y+1=\frac{9}{2}(x-1)$ *or* $y=\frac{9}{2}x-\frac{11}{2}$ *or* $y=4.5\,x-5.5$	*1 points*
7	1. (a,b) U (d,e) U (g,h) 2. x = 0 3. x = b, h	*3 points*
8	1. $\left(-\infty,-\frac{2}{3}\right)U\ (0,\infty)$ 2. $\left(-\frac{2}{3},\frac{4}{9}e^{-2}\right)$	*2 points*
9	a = 1/4, b = -9/4	*1 point*
10	$x=\sqrt{5}$	*1 point*
11	40	*1 point*
12	$9\pi+72$	*1 point*
13	7/48	*1 point*
14	$x=0,\pi,\frac{\pi}{6},\frac{11\pi}{6}$	*1 point*
15	-875	*1 point*
16	-15/16	*1 point*
17	1. $2\pi+5$ 2. $2\pi+15$	*2 points*
18	e	*1 point*

Final Showdown (Answers)—Played at Buzzers—NO CALCULATOR	
1	$6\pi x$
2	*-1/8*
3	*Y = -1 (it's a horizontal line, the tangent line is vertical since y' is undefined at (-2, -1)*
4	$-\csc^2\theta$ *(given limit is the definition of the derivative of tangent)*
5	$x = \frac{\pi}{4}, \frac{5\pi}{4}$
TB	*V(3) = -1 m/s (it's the slope of the given fxn at t = 3, which is –1/1 = -1.)*

Calculus Competition March 1, 2019

Exam Format and Timeline

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11:25-11:50AM	*Auditorium*	*Final Showdown (with buzzers)--Top three teams (scoring is explained below) will face off in front of all the other teams to determine the 1st, 2nd, and 3rd place.*
NOON	*Auditorium*	*Pizza party in the cafeteria for all participants--If students want pizza/beverage, please have them bring $5...we'll order loads of pizza and the kids can hang out and eat before you all leave. If you have a student who wants pizza but can't swing the $5, let me know and we'll work something out.*

At the conclusion of the final showdown round, all team and individual winners will be recognized. Awards will be presented.

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SCORING: To determine the top three teams for the "final showdown", we will sum the team round score with the top three individual scores on the team (this way we are not penalizing teams with only 3 students.) The three teams with the highest scores will be in the showdown. IF there is a tie that yields more than 3 teams for the showdown, the tie will be broken as follows: only the top scorer on the individual exams will be counted...if there is still a tie, then only the top one scorers on each team will be counted...if there is still a tie, only the top scorers on each team will be counted...if there is still a tie, more than 3 teams will participate in the showdown.

Each team will consist of 3 to 5 students. Schools may bring multiple teams.

Name:	*School:*	*Team#:*

Individual Round: (Non-Calculator)

1 *Let y =f(x) be a solution to the differential equation:* $\frac{dy}{dx} = \frac{\cos x}{y}$ *and let* $f(0) = -4$. *Find* $f\left(\frac{\pi}{6}\right)$.

A. $-\sqrt{17}$

B. $-4\sqrt{e}$

C. $-\frac{7}{2}$

D. $4\sqrt{e}$

E. $\sqrt{17}$

2 Information about two continuous, differentiable functions, f(x) and g(x), are given in the table below. Given that h(x) = f(x)/g(x), what is the value of h'(2)?

x	f(x)	f '(x)	g(x)	g '(x)
1	2	6	1	0
2	3	-5	-2	1
3	-1	2	4	10

A. -5

B. -13/4

C. -7/4

D. 7/4

E. 13/4

3 Hawkins Lab is leaking radioactive ooze at a rate of R(t), in gallons per minute. Select values for R(t) are provided in the table below. Using a left Riemann sum as an estimate, what is the least amount of ooze that has leaked out of the lab during the 8-minute period?

t (minutes)	0	1	2	4	5	8
R(t) (gallons/minute)	3	2	7	10	5	2

A. 18 gallons
B. 24 gallons
C. 27 gallons
D. 29 gallons
E. 44 gallons

4	Fletch, the amazing super baby, is crawling along the x-axis. His position, F(t), can be modeled by the following equation for all t ≥ 0. $$F(t) = t^3 - \frac{7}{2}t^2 + 2t + 4$$ What is Fletch's acceleration the first time he is at rest? A. -7 B. -5 C. 0 D. 5 E. 7

5	$F(x) = \int_4^{x^2+1} \ln t \, dt$, find F'(x). A. $\frac{1}{x^2+1} - \frac{1}{4}$ B. $2x \ln\lvert x^2+1\rvert - \ln 4$ C. $\frac{2x}{x^2+1}$ D. $2x \ln(x^2+1)$ E. $\ln\lvert x^2+1\rvert$
6	$f(x) = e^{2x}\cos(3x)$ which of the following accurate describes the function at x = 0? A. positive, decreasing, concave up B. negative, decreasing, concave up C. positive, increasing, concave up D. negative, increasing, concave down E. positive, increasing, concave down

7	$$If\ \frac{d}{dx}f(x) = g(2x)\ \ and \frac{d}{dx}g(x) = f(x^2), then\ \frac{d^2}{dx^2}f(x^5) =$$ A. $f(4x^{10})$ B. $20x^3 f(4x^{10})$ C. $5x^4 g(2x^5)$ D. $20x^3 g(2x^5) + 5x^4 f(4x^{10})$ E. $20x^3 g(2x^5) + 50x^8 f(4x^{10})$
8	$$\int x^2 \sqrt[3]{x+2}\, dx$$ A. $\frac{3}{10}(x+2)^{\frac{10}{3}} - \frac{12}{7}(x+2)^{\frac{7}{3}} + 3(x+2)^{\frac{4}{3}} + C$ B. $\frac{2}{7}(x+2)^{\frac{7}{2}} - \frac{8}{5}(x+2)^{\frac{5}{2}} + \frac{8}{3}(x+2)^{\frac{3}{2}} + C$ C. $\frac{3}{10}(x+2)^{\frac{10}{3}} + 3(x+2)^{\frac{4}{3}} + C$ D. $\frac{3}{10}x^{\frac{7}{3}} + \frac{2}{3}x^3 + C$ E. $\frac{1}{4}x^3(x+2)^{\frac{4}{3}} + C$

SCHOOL:	***TEAM #:***

Question 1: (3 minutes)

$$\int \frac{7x^3}{\cos^2(x^4)}\,dx$$

SCHOOL:	***TEAM #:***

<u>Question 2: (3 minutes)—Worth 2 points</u>

A particle moves along the x-axis so that its velocity at time t is given by

$v(t) = e^t t^2$.

1. Write an expression for the acceleration, a(t), of the particle at time t.

2. On what time interval is the particle slowing down?

SCHOOL:	***TEAM #:***

<u>Question 3: (3 minutes)</u>

Given $f(x) = \tan^2(5x)$, *find* $f'\left(\frac{\pi}{20}\right)$.

Fully simplify your answer.

SCHOOL:	***TEAM #:***

Question 4: (3 minutes)—Worth 2 points

The function, f(x), the first derivative, f'(x), and the second derivative, f''(x), are continuous. f(x) has exactly three zeros. f'(x) has exactly two zeros and f''(x) has exactly one zero . Selected values of the function and both derivatives are given in the table below.

x	-2	-1	0	1	2	3	4	5	6
f(x)	-5	-3	-1	2	1	0	-1	2	4
f'(x)	4	2	1	0	-1	-2	0	1	3
f''(x)	-6	-4	-2	-1	0	2	3	6	7

1. What are the coordinates of the relative minimum of f(x)?

2. If the domain of f(x) is all real numbers on what interval(s) is f(x) concave down and increasing?

SCHOOL:	***TEAM #:***

<u>Question 5: (3 minutes)</u>

What are the ***<u>coordinates</u>*** *of the point in the 3rd quadrant on the curve* $y = 1 - x^2$ *that is closest to (0, -2)?*

SCHOOL:	***TEAM #:***

<u>Question 6: (3 minutes)</u>

What is the equation of the line **<u>tangent</u>** to the curve below at (1, 2)?

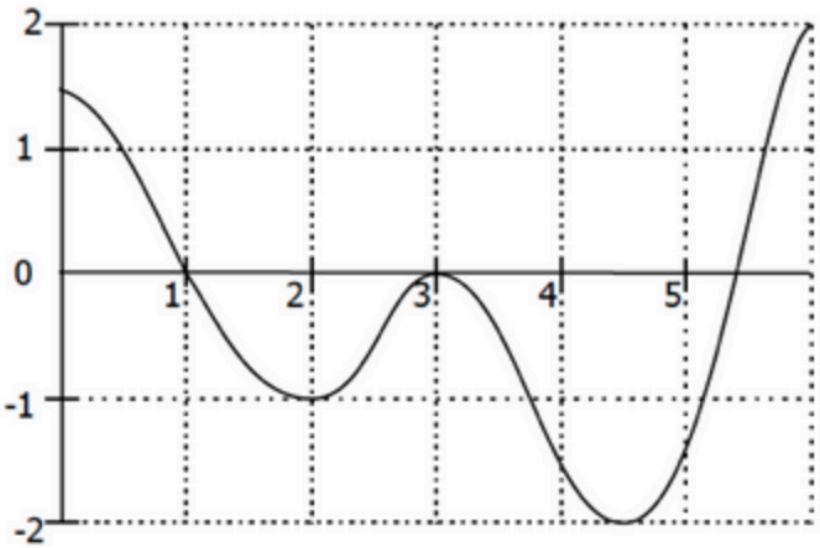
$$3x^4 - x^2y = 1$$

SCHOOL:	***TEAM #:***

<u>Question 7</u>: (3 minutes)—Worth 3 points

Below is the graph of f '(x) on the open interval (0, 6.) f'(x) = 0 at x = 1, 3, and 5.4. f '(x) is level and has relative extrema at x = 0, 2, 3, and 4.5.

<u>Graph of y = f '(x)</u>



1. *On what interval(s) is f(x) increasing and concave down?*

2. *For what x value does f(x) have a relative minimum?*

3. *For what x value(s) does f(x) have a point of inflection?*

SCHOOL:	***TEAM #:***

<u>Question 8</u>: (3 minutes)—Worth 2 points

$$f(x) = e^{-2x}x^3$$

1. *On what intervals of x is f increasing?*

2. *What are the **<u>coordinates</u>** of the absolute maximum of the graph?*

SCHOOL:	***TEAM #:***

<u>Question 9</u>: (3 minutes)

$$f(x) = \begin{cases} 2ax + 3, & x < 0 \\ a\cos x + b\sin x, & x \geq 0 \end{cases}$$

If f(x) is continuous and differentiable for all real numbers, x, what is the value of a+b?

SCHOOL:	***TEAM #:***

<u>Question 10:</u> ***(3 minutes)***

$$f(x) = \frac{24}{x} + 5$$

Find all values, c, in [1, 4] guaranteed by the Mean Value Theorem for derivatives.

SCHOOL:	***TEAM #:***

<u>Question 11: (3 minutes)</u>

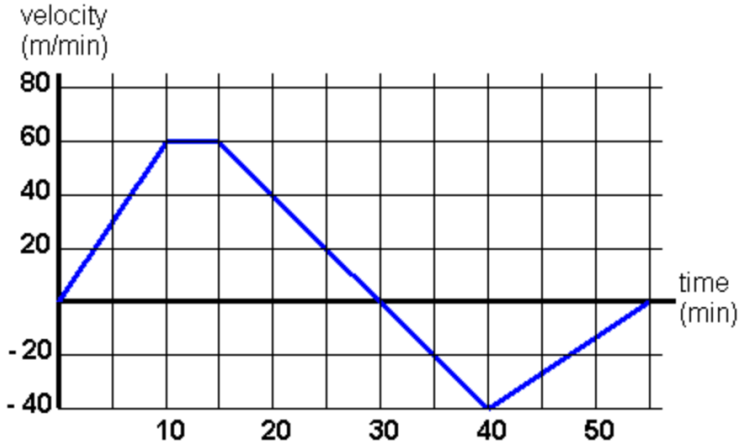
Given: $f(x) = -9x^2 - 4x + 7$

Find the average value of f(x) on [1, 3.]

SCHOOL:	***TEAM #:***

<u>Question 12: (3 minutes)</u>

The velocity, v(t), of a particle (in meters/minute) moving along the x-axis is given by the graph below. It consists of four line segments.
Assume all changes in the position graph occur at points with integer coordinates (in case it's difficult to see the grid.)



With correct units, what the is the total distance traveled by the particle from t = 0 minutes to t = 55 minutes?

SCHOOL:	***TEAM #:***

<u>Question 13: (3 minutes)</u>

Evaluate the integral. Fully simplify your answer.

$$\int_{0}^{\frac{\pi}{4}} sec^2(3x)\, tan(3x)\, dx$$

SCHOOL:	***TEAM #:***

<u>Question 14: (3 minutes)</u>

For what x value(s) does f(x) have a horizontal tangent line in $[0,2\pi)$?

$$f(x) = \cos^2 x + \sqrt{3}cosx$$

SCHOOL:	***TEAM #:***

<u>Question 15:</u> ***(3 minutes)***

What is the minimum value of f(x) on [-3, 3]?

$$f(x) = -x^3 + 3x + 4$$

SCHOOL:	***TEAM #:***

<u>Question 16</u>: (3 minutes)

Evaluate the limit.

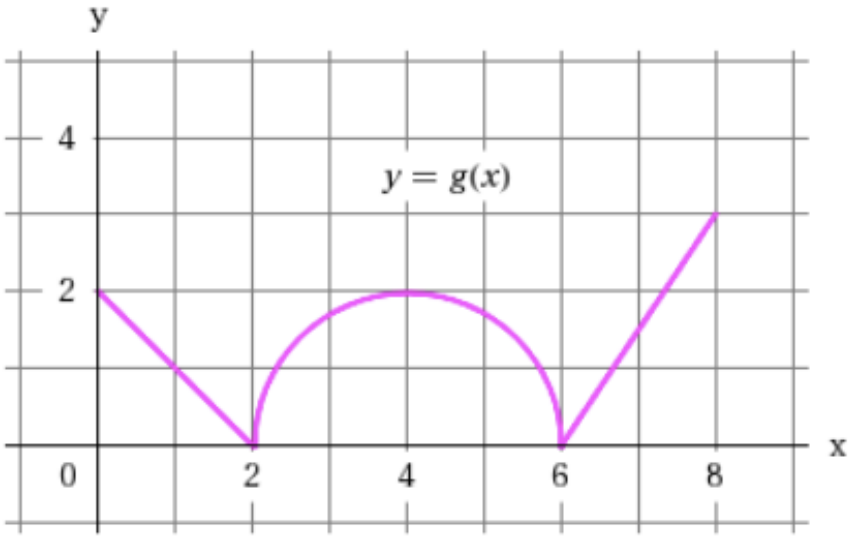
$$\lim_{k \to 0} \frac{\frac{10}{2+k} - 5}{k}$$

SCHOOL:	***TEAM #:***

<u>Question 17: (3 minutes)—Worth 2 points</u>

The graph of g(x), shown below, consists of a line segment from x = 0 to x = 2, a semicircle from from x = 2 to x = 6, and a line segment from x = 6 to x = 8.

All answers must be fully simplified to receive credit.



1. *What is the value of* $\int_6^0 g(x)dx$*?*

2. *What is the value of* $\int_4^8 (g(x) + x)\, dx$*?*

SCHOOL:	***TEAM #:***

Question 18:

Evaluate the limit.

$$\lim_{x \to 0} x^x$$

FINAL SHOWDOWN QUESTION 1:

$$g(x) = \frac{\sqrt{e^3+3e^2+5e-10}}{e^6 \cos \pi e} + 6e^2 \cos \frac{x}{2}$$

Find g'(x).

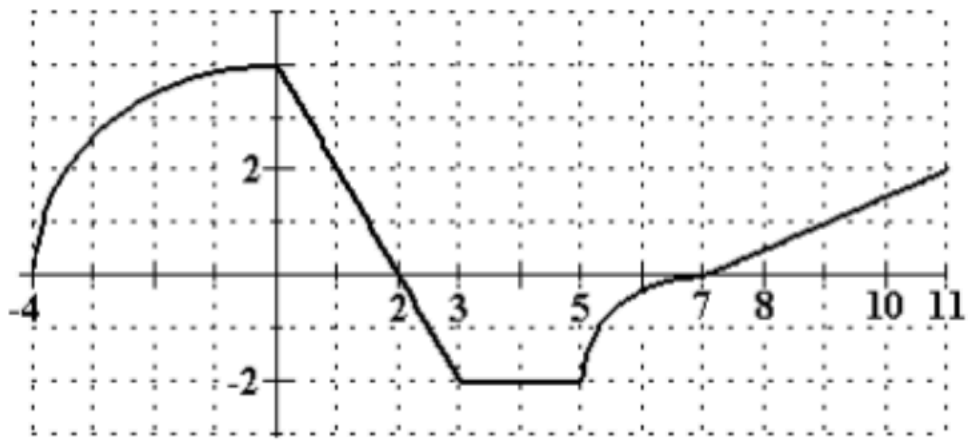
FINAL SHOWDOWN QUESTION 2:

Evaluate the limit:

$$\lim_{x \to 5^+} \frac{|5-x|}{8x-40}$$

FINAL SHOWDOWN QUESTION 3: *The graph of f(x) is given below. If consists of two quarter circles and three line segments.*

Graph of y = f(x)



Give the exact solution to the following integral:

$$\int_{-4}^{11} f(x) + 3\, dx$$

FINAL SHOWDOWN QUESTION 4:

Evaluate the limit.

$$\lim_{h \to 0} \frac{\frac{1}{\cos(\theta + h)} - \frac{1}{\cos(\theta)}}{h}$$

FINAL SHOWDOWN QUESTION 5:

$$f(x) = e^x \sec x \qquad [0, 2\pi)$$

For what value(s) of x does f (x) have a horizontal tangent line?

TIE-BREAK QUESTION:

Give the equation of the line normal to the curve $xy^2 + 2x^3y + 1 = 0$ *at x = 1.*

Individual Exam (Answers)—NO CALCULATOR		
1	*A*	
2	*D*	
3	*E*	
4	*B*	
5	*D*	
6	*E*	
7	*E*	
8	*A*	
Team Round (Answers)—NO CALCULATOR		
1	$\frac{7}{4}\tan(x^4) + C$	*1 point*
2	1. $a(t) = te^t(t+2) = t^2e^t + 2te^t$ 2. $(-2,0)$	*2 points*
3	*20*	*1 point*
4	1. $(4,-1)$ 2. $(-\infty, 1)$	*2 points*
5	$\left(-\sqrt{\frac{5}{2}}, -\frac{3}{2}\right)$	*1 points*
6	$y - 2 = 8(x-1)$ OR $y = 8x - 6$	*1 points*
7	1. *(0, 1)* 2. *x = 5.4* 3. *x = 2, 3, 4.5*	*3 points*
8	1. $(-\infty, 0) \cup \left(0, \frac{3}{2}\right)$ 2. $\left(\frac{3}{2}, \frac{27}{8}e^{-3}\right)$	*2 points*
9	*9*	*1 point*
10	x = 2 (not -2, because that is not in the interval)	*1 point*
11	-40	*1 point*
12	*1550 meters*	*1 point*
13	1/6	*1 point*
14	$x = 0, \pi, \frac{5\pi}{6}, \frac{7\pi}{6}$	*1 point*
15	-14	*1 point*
16	-5/2 (or -10/4 or -2.5)	*1 point*
17	1. $-2 - 2\pi$ 2. $\pi + 27$	*2 points*
18	1	*1 point*
Final Showdown (Answers)—Played at Buzzers—NO CALCULATOR		
1	$-3e^2 \sin\left(\frac{x}{2}\right)$	
2	*1/8*	
3	$5\pi + 44$	
4	$sec\theta tan\theta$	
5	$x = \frac{3\pi}{4}, \frac{7\pi}{4}$	
TB	*Y = -1*	