APAB S.Hogan
Chapter Homework Assignment
Name: ________________________________

APAB Competition Review HW

Due on ___________ (day AFTER competition) at the end of class.

Directions: Use this sheet as a cover sheet for your homework assignment. If you do not staple this cover sheet to the front of your assignment, you will receive a zero. You may submit your HW early but NO LATE HW will be accepted.

Assignment:

2013 Calculus Competition
2014 Calculus Competition
(Complete ALL problems)

REMEMBER, THIS CALCULUS COMPETITION WILL COUNT AS AN EXAM FOR YOU. YOUR TEAM SCORE WILL BE ADDED TO YOUR INDIVIDUAL SCORE.

Grading:

Item	Possible Points	Score
All Problems Completed	2	
Problems Labeled and in Numerical Order, Page Numbers are Labeled	2	
First Random Problem (correct with sufficient work)	4	
Second Random Problem (correct with sufficient work)	4	
Third Random Problem (correct with sufficient work)	4	
Fourth Random Problem (correct with sufficient work)	4	
TOTAL	20	

Name:	School:	Team:

Individual Round: (Non-Calculator)

1	$y=(\sin x)^x$, find y'. A. $y' = x\cot x + \ln(\sin x)$ B. $y' = x(\sin x)^{x-1}$ C. $y' = x(\sin x)^{x-1} * \cos x$ D. $y' = (\sin x)^x[x\cot x + \ln(\sin x)]$ E. $y' = \ln(\sin x) * (\sin x)^x$
2	$F(x) = \int_{\tan x}^{8} t^3\, dt$ F'(x) = A. $F'(x) = 3\tan^2 x$ B. $F'(x) = 3\tan^2 x \sec^2 x$ C. $F'(x) = -3\tan^2 x \sec^2 x$ D. $F'(x) = -\tan^3 x \sec^2 x$ E. $F'(x) = \tan^3 x \sec^2 x$
3	What is the area of the region bound by $y = x^3 - 2x^2 - 4x + 8$ and the x-axis? A. -64/3 B. -44/3 C. 20/3 D. 44/3 E. 64/3
4	A particle, initially at rest, has an acceleration of a(t) = 12t – 24. What is the total distance traveled by the particle from t = 0 to t = 5? A. -64 B. -50 C. 50 D. 64 E. 78
5	A 25 ft ladder is leaning against a building. The base of the ladder is being pulled away from the building at a rate of 2 ft/sec. At the moment when the base of the ladder is 7 feet from the building, at what SPEED is the top of the ladder sliding DOWN the wall? A. -14/25 ft/sec B. -7/12 ft/sec C. -7/24 ft/sec D. 7/12 ft/sec E. 7/24 ft/sec

6 Using the table provided use the trapezoidal approximation (with 3 trapezoids) to estimate the area under the curve of f(x) from x = 0 to x = 8.

x	f(x)
0	2
3	1
5	7
8	10

A. 28
B. 112/3
C. 38
D. 42
E. 76

7 $\lim_{n\to 0} \dfrac{\tan(x+n)-\tan x}{n} =$

A. 0
B. undefined
C. $\tan x$
D. $\sec^2 x$
E. $-\ln|\cos x|$

8 A farmer has 48 feet of fencing. He would like to use all of this fencing to create the biggest possible rectangular grazing area for his horses but plans to construct the enclosure so that one side of the rectangle is the existing wall of a barn. What is the maximal area that he can create?

A. 12 ft^2
B. 24 ft^2
C. 144 ft^2
D. 256 ft^2
E. 288 ft^2

9 What best describes the function $f(x) = x^3 - 3x^2 - 45x + 135$ on the interval $1 < x < 2$.

A. Decreasing, concave up, positive
B. Decreasing, concave up, negative
C. Decreasing, concave down, negative
D. Increasing, concave up, positive
E. Increasing, concave down, negative

10 What is the minimum value of f(x) = 2cosx – x on the interval [0, 2pi)?

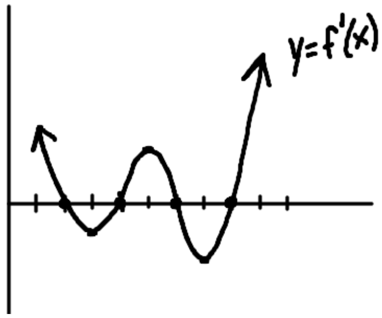
A. $\dfrac{7\pi}{6}$ B. $\sqrt{3}-\dfrac{7\pi}{6}$ C. $\sqrt{3}-\dfrac{11\pi}{6}$

D. $-\sqrt{3}-\dfrac{7\pi}{6}$ E. -1

11	If $\frac{d}{dx}f(x)=g(x^4)$ and $\frac{d}{dx}g(x)=f(2x)$, then $\frac{d^2}{dx^2}f(x^3)=$ A. $3x^2g(x^{12})$ B. $6xg(x^{12})+36x^{13}f(2x^{12})$ C. $36x^{13}f(2x^{12})$ D. $6xg(x^{12})+36x^{13}f(x^{36})$ E. $f(2x^{12})$
12	If $h(x)=f(g(x))$, use the given table to find h'(2). <table><tr><th>x</th><th>f(x)</th><th>g(x)</th><th>f'(x)</th><th>g'(x)</th></tr><tr><td>1</td><td>0</td><td>4</td><td>1</td><td>-3</td></tr><tr><td>2</td><td>6</td><td>3</td><td>-2</td><td>2</td></tr><tr><td>3</td><td>1</td><td>0</td><td>5</td><td>7</td></tr></table>A. -4 B. -2 C. 2 D. 6 E. 10
13	$f(x)=\frac{x^4+3x^2}{\sqrt{2x+3}}$ Find f'(x). A. $\frac{4x^3+6x}{\sqrt{2}}$ B. $\frac{\sqrt{2x+3}\left[(4x^3+6x)-(x^4+3x^2)\right]}{2x+3}$ C. $\frac{7x^4+12x^3+9x^2+18x}{(2x+3)^{3/2}}$ D. $\frac{\left[(4x^3+6x)-(x^4+3x^2)\right]}{\sqrt{2x+3}}$ E. $\frac{-x^4+4x^3-3x^2-6x}{2x+3}$
14	What values, c, satisfy the Mean Value Theorem for derivatives for f(x) = x^3 – 5x + 13 on the open interval (0, 2)? A. $\frac{2}{\sqrt{3}}$ and $-\frac{2}{\sqrt{3}}$ B. $-\frac{2}{\sqrt{3}}$ only C. $\frac{2}{\sqrt{3}}$ only D. 4/3 and -4/3 E. 4/3, -4/3, and 1

15	$f(x) = \frac{10}{\sqrt{x}}$ What is the average value of f(x) on the interval [4, 9]? A. 25/6 B. 4 C. 25/3 D. 2ln(9/4) E. 20
16	$\int \sqrt[3]{\cos x} \sin x dx$ A. $\frac{3}{4}(\cos x)^{3/4} + C$ B. $-\frac{3}{4}(\cos x)^{4/3} + C$ C. $\frac{3}{4}(\cos x)^{4/3} + C$ D. $-\frac{2}{3}(\cos x)^{3/2} + C$ E. $\frac{2}{3}(\cos x)^{3/2} + C$
17	$\int \frac{dx}{\sqrt{49 - 4x^2}}$ A. $\frac{1}{2}\arcsin\left(\frac{2x}{7}\right) + C$ B. $\frac{1}{7}\arcsin\left(\frac{2x}{7}\right) + C$ C. $\arcsin\left(\frac{x}{7}\right) + C$ D. $2\sqrt{49 - 4x^2} + C$ E. $2\ln(49 - 4x^2) + C$
18	A particle moves along the x-axis such that it's position is modeled by the equation $x(t) = t^3 - 6t^2 - 15t + 3$ for all $t \geq 0$. What is the velocity of the particle when its acceleration is equal to 0? A. -43 B. -27 C. -18 D. 0 E. 18

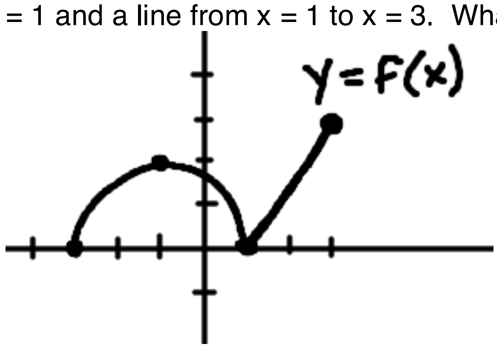
19 This is the graph of f'(x).



On which of the following intervals is f(x) concave down AND increasing on the entire interval?

A. (2, 3)
B. (2, 4)
C. (3, 5)
D. (5, 6)
E. (5, 7)

20 The graph of f(x), shown below, consists of a semicircle of radius 2 from x = 3 to x = 1 and a line from x = 1 to x = 3. What is the value of $\int_{-1}^{3}(f(x)+2)\,dx$



A. $\pi+3$
B. $\pi+11$
C. $2\pi+3$
D. $2\pi+11$
E. $4\pi+11$

Free Response: The acceleration of a particle moving along the x-axis is given by $a(t) = 6t - 4$ for all $t > 0$. At time $t = 1$, the position of the particle is -2 and the particle is at rest.

A. During what time interval(s) is the particle moving to the left?

B. What is the total distance traveled by the particle in the interval [0, 4]?

C. During what times intervals is the **speed** of the particle decreasing?

Team Round Questions

Question 1: (3 minutes)

$$\int 4x^2 \cos(x^3 + 3)\,dx =$$

Question 2: (3 minutes)

A particle moves along the x-axis so that its velocity at time t (where $0 \leq t < 2$) is given by $v(t) = \dfrac{1}{\sqrt{4-t^2}}$.

Given that $x(1) = \dfrac{\pi}{6} + 2$ …

A. Write an expression for the position, x(t), of the particle at time t.

B. Write an expression for the acceleration, a(t), of the particle at time t.

Question 3: (3 minutes)

Given $f(x) = e^{x^2+5x}$, find $f'(2)$.

Question 4: (3 minutes)

The derivative, g'(x), and the second derivative, g"(x), of a function, g(x), are continuous and each have exactly two zeros. Selected values of both derivatives are given in the table below.

x	- 4	- 3	- 2	- 1	0	1	2	3	4
g'(x)	- 3	- 2	0	1	2	0	1	3	5
g"(x)	2	0	- 1	- 3	0	2	4	5	6

A. If the domain of g(x) is the set of all real numbers, on what interval(s) is g(x) BOTH increasing and concave up?

B. What is the x-coordinate(s) of the point(s) of inflection?

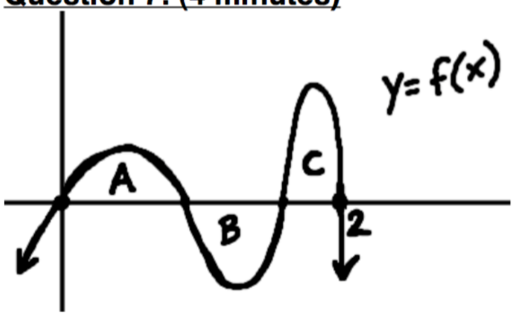
Question 5: (4 minutes)

What are the coordinates of the point on the curve $y = 2x - 3$ that is closest to $(3, 0)$?

Question 6: (4 minutes)

What is the equation of the line tangent to the curve $3x^2 - 2y^3x^2 = 3 + y^4$ at $(-2, 1)$?

Question 7: (4 minutes)



The regions A, B, and C in the figure above are bounded by the graph of the function f and the x-axis. If the area of each region is 4, what is the value of

$$\int_0^2 (f(x) - 6)\,dx$$

Question 8: (4 minutes)

$$f(x) = x + \frac{4}{x}$$

On what intervals of x is f decreasing?

Question 9: (4 minutes)

$$f(x) = \begin{cases} 3x^2, x \le 1 \\ ax - 2b, x > 1 \end{cases}$$

If f(x) is continuous and differentiable for all real numbers, x, what is the value of a + b?

Individual Exam (Answers)	
1	D
2	D
3	E
4	E
5	D
6	C
7	D
8	E
9	A
10	D
11	B
12	E
13	**C. Answer looks like it lines up with B, but it is the 3rd answer, so it's C.**
14	C
15	B
16	B
17	A
18	B
19	D
20	B
21FR	A. (1/3, 1) B. 980/27 C. [0, 1/3) U (2/3, 1)
Team Round (Answers)	
1	$(4/3)\sin(x^3+3)+C$
2	A. $x(t) = \arcsin(t/2) + 2$ B. $t/(4-t^2)^{(3/2)}$
3	$9e^{14}$
4	A. (0,1)U(1, inf) B. x=-3, x = 0
5	(9/5, 3/5)
6	Y – 1 = (-1/7)(x + 2)
7	-8
8	(-2, 0)U(0, 2)
9	15/2

Name:	School:	Team:

Individual Round: (Non-Calculator)

1	$y = x^{\tan x}$, find y'. A. $y' = \tan x \bullet x^{\tan x - 1}$ B. $y' = \ln x \bullet x^{\tan x}$ C. $y' = x^{\tan x} \tan x + x^{\tan x} \ln x \bullet \sec x$ D. $y' = x^{\tan x - 1} \tan x + x^{\tan x} \ln x \bullet \sec^2 x$ E. $y' = \frac{\tan x}{x} + \ln x \bullet \sec^2 x$
2	$F(x) = \int_{e^x}^{3} t^3 - 2t^2 \, dt$ F'(x) = A. $-e^{4x} + 2e^{3x}$ B. $e^{4x} - 2e^{3x}$ C. $e^{3x} - 2e^{2x}$ D. $-e^{3x} + 2e^{2x}$ E. $-e^{4x} + 2e^{3x} - 9$
3	What is the area of the region bound by $y = x^3 - x^2 + 8x - 8$, $x = 2$,and the x-axis? A. $\frac{4}{3}$ B. $\frac{65}{12}$ C. 8 D. 12 E. 16
4	A particle, initially at rest, has an acceleration of $a(t) = 18t - 6$. What is the total distance traveled by the particle from t = 0 to t = 2? A. $\frac{4}{3}$ B. $\frac{104}{9}$ C. $\frac{112}{9}$ D. 12 E. $\frac{116}{9}$

5	A 26 ft ladder is leaning against a building. The base of the ladder is being pulled away from the building at a rate of 3/2 ft/sec. At the moment when the base of the ladder is 10 feet from the building, at what SPEED is the top of the ladder sliding down the wall? A. $-\frac{5}{8}\,ft/s$ B. $-\frac{5}{12}\,ft/s$ C. $\frac{5}{12}\,ft/s$ D. $\frac{5}{8}\,ft/s$ E. $\frac{323}{12}\,ft/s$
6	Use a trapezoidal approximation (n = 3) to estimate the area under the curve of $f(x)=x^2-3x+2$ from $x=0$ to $x=6$. A. 30 B. 31 C. 34 D. 62 E. 68
7	$\lim\limits_{n\to 0}\dfrac{\frac{3}{(x+n)^2}-\frac{3}{x^2}}{n}=$ A. undefined B. $-\frac{3}{x}$ C. 0 D. $-\frac{3}{x^2}$ E. $-\frac{6}{x^3}$
8	A farmer has 60 feet of fencing. He would like to use all of this fencing to create the biggest possible rectangular pen for his pigs. He plans to construct the enclosure so that one side of the rectangle is the existing wall of a barn. What is the maximal area that she can create? A. $15\,ft$ B. $30\,ft$ C. $225\,ft^2$ D. $450\,ft^2$ E. $900\,ft^2$

9	What best describes the function $f(x)=x^3-9x^2+24x-216$ on the interval $4<x<5$? A. positive, increasing, and concave down B. negative, increasing, and concave up C. positive, increasing, and concave up D. negative, decreasing, and concave down E. negative, decreasing, and concave up
10	What is the maximum value of $f(x)=x^3-3x^2+5$ on the interval $[-1,4]$? A. 0 B. 4 C. 5 D. 21 E. 24
11	If $\frac{d}{dx}f(x)=g(x^3)$ and $\frac{d}{dx}g(x)=f(4x)$, then $\frac{d^2}{dx^2}f(x^2)=$ A. $2g(x^6)+12x^6f(4x^6)$ B. $6xg(x^3)+9x^4f(4x^3)$ C. $f(4x^6)$ D. $24x^5f(4x^6)$ E. $2xf(4x^6)$
12	If $h(x)=\frac{f(x)}{g(x)}$, $h'(6)=5$, and $g(x)=3$, what is $f'(6)=?$ A. 0 B. 2 C. 15 D. 17 E. The given information is not sufficient to find f'(6).
13	$f(x)=\frac{x^2-8x}{\sqrt{x^3+2}}$ Find f'(x). A. $(4x-16)\sqrt{x^3+2}$ B. $\frac{(4x-16)\sqrt{x^3+2}}{3x^2}$ C. $\frac{7x^4-40x^3+8x-32}{2(x^3+2)^{3/2}}$ D. $\frac{-x^4+16x^3+4x-16}{2(x^3+2)^{3/2}}$ E. $\frac{x^4+8x^3+8x-32}{2(x^3+2)^{3/2}}$

14	What values, c, satisfy the Mean Value Theorem for derivatives for $f(x)=\dfrac{3}{x+2}$ on the open interval (0, 3)? A. $-2+\sqrt{10}$ B. $2+\sqrt{10}$ C. $-2-\sqrt{10}$ D. 98 E. $-2\pm\sqrt{10}$
15	$f(x)=\dfrac{x-3}{x}$ What is the average value of f(x) on the interval [1, 4]? A. $3-3\ln 4$ B. $\ln\left(\dfrac{e}{4}\right)^3$ C. $\ln\left(\dfrac{e}{4}\right)$ D. $-\dfrac{3}{2}$ E. $-\dfrac{1}{2}$
16	$\int\sqrt{\cot x}\csc^2 x\,dx$ A. $\dfrac{2}{3}\sqrt{\cot^3 x}+C$ B. $-\dfrac{2}{3}\sqrt{\cot^3 x}+C$ C. $\dfrac{3}{2}\sqrt{\cot^3 x}+C$ D. $-\dfrac{3}{2}\sqrt{\cot^3 x}+C$ E. $\dfrac{1}{2}\cot^2 x+C$

17 $\int \frac{2x-5}{\sqrt{4x-x^2}}\,dx$

A. $-18\sqrt{4x-x^2}+C$

B. $-\frac{27}{2}\left(4x-x^2\right)^{3/2}+C$

C. $(2x-5)\arcsin\left(\frac{x-2}{2}\right)+C$

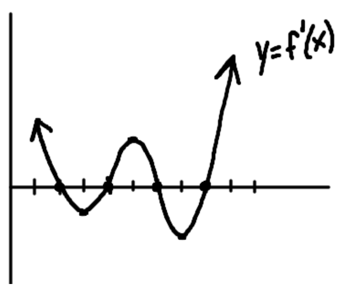
D. $-2\sqrt{4x-x^2}+\arcsin\left(\frac{x-2}{2}\right)+C$

E. $-2\sqrt{4x-x^2}-\arcsin\left(\frac{x-2}{2}\right)+C$

18 A particle moves along the x-axis such that it's position is modeled by the equation $x(t) = 2t^3 - 5t^2 + 3t + 4$ for all $t \geq 0$. What is the velocity of the particle when its acceleration is equal to 0?

A. -10 B. $-\frac{7}{6}$ C. $\frac{5}{6}$ D. 3 E. 4

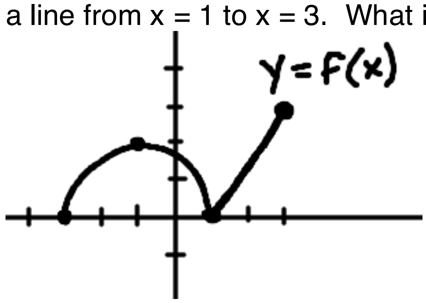
19 This is the graph of f'(x).



On which of the following intervals is f(x) concave down AND decreasing on the entire interval?

A. (2, 4) B. (3, 4) C. (5, 7) D. (6, 7) E. (6, 8)

20 The graph of f(x), shown below, consists of a semicircle of radius 2 from x = -3 to x = 1 and a line from x = 1 to x = 3. What is the value of $\int_{3}^{-3}(f(x)-2)\,dx=$



A. $-4\pi+9$ B. $-2\pi+9$ C. $2\pi-9$ D. $4\pi-9$ E. $-2\pi+18$

Name:	School:	Team:

21	Free Response: The acceleration of a particle, launched with an initial velocity of 2, moving along the x-axis is given by a(t) = 6t – 5 for all t $\geq$ 0. At time t = 2, the position of the particle is 8. A. At what times if the particle at rest? B. Write an expression for the position, x(t), of the particle at time t. C. During what time intervals is the particle moving to the right? D. During what times intervals is the particle **speeding up**?

Team Round Questions

Question 1: (3 minutes)

$$\int -6x^2\sqrt{x^3+5}\,dx$$

Question 2: (3 minutes)

$f(x) = 2xe^{-x}$

On what intervals of x is f decreasing?

Question 3: (3 minutes)

$$f(x) = \begin{cases} 2bx, x < 3 \\ ax^2 + 3, x \geq 3 \end{cases}$$

If f(x) is continuous and differentiable for all real numbers, x, what is the value of a – b?

Question 4: (3 minutes)

Given $f(x) = \tan^5(5x)$, find $f'\left(\frac{\pi}{30}\right)$.

Question 5: (3 minutes)

What is the equation of the line normal to the curve $12x^2 - 4xy^2 = 3x - y^3$ at (1, 3)?

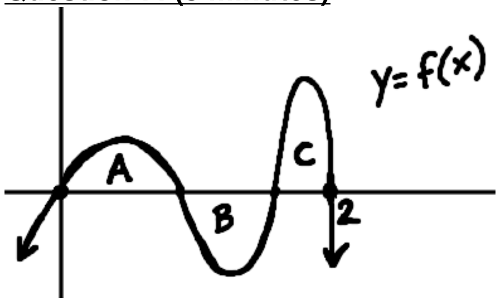
Question 6: (3 minutes)

The derivative, g'(x), and the second derivative, g"(x), of a function, g(x), are continuous and each have exactly two zeros. Selected values of both derivatives are given in the table below.

x	- 4	- 3	- 2	- 1	0	1	2	3	4
g'(x)	- 3	- 2	0	1	2	0	1	3	5
g"(x)	2	0	- 1	- 3	0	2	4	5	6

If the domain of g(x) is the set of all real numbers, on what interval(s) is g(x) BOTH concave up AND decreasing?

Question 7: (3 minutes)



The regions A, B, and C in the figure above are bounded by the graph of the function f and the x-axis. If the area of each region is 4, what is the value of $\int_{2}^{0}(f(x)-3)\,dx$?

Question 8: (3 minutes)

A particle moves along the x-axis so that its velocity at time t (where $0 \leq t < 2$) is given by $v(t)=\sqrt[3]{t^2-6t}$.

Write an expression for the acceleration, a(t), of the particle at time t.

Question 9: (4 minutes)

What are the coordinates of the point on the curve y = 3x – 1 that is closest to (2, 0)?

TIE BREAK QUESTION 1:

$\int \frac{x}{\sqrt{x+5}}dx =$

TIE BREAK QUESTION 2:

A particle's velocity is given by the equation $v(t)=3t^2-8t+5$
What is the total distance traveled by the particle for $0 \leq t \leq 5$?

TIE BREAK QUESTION 3: $g(x)=3\pi^4-\sin\pi+\sqrt[3]{2\pi+e^{3\pi}}$ find g'(x).

TIE BREAK QUESTION 4:

At noon, Car A begins driving due east away from Mathville at a rate of 60mph. One hour later, Car B leaves Calcburg (located 100 miles due north of Mathville) and begins driving due south towards Mathville at a rate of 50mph. How fast is the distance between Car A and Car B changing a 2PM?

TIE BREAK QUESTION 5: $f(x)=\sin^5(e^{3x})$ find f'(x).

Individual Exam (Answers)	
1	D
2	A
3	B
4	E
5	D
6	C
7	E
8	D
9	B
10	D
11	A
12	C
13	E
14	A
15	C
16	B
17	E
18	B
19	D
20	B
21FR	A. $t = \frac{2}{3}$ and $t = 1$ B. $x(t) = t^3 - \frac{5}{2}t^2 + 2t + 6$ C. $[0, \frac{2}{3}) \cup (1, \infty)$ D. $\left(\frac{2}{3}, \frac{5}{6}\right) \cup (1, \infty)$
Team Round (Answers)	
1	$-\frac{4}{3}(x^3 + 5)^{3/2} + C$
2	$(1, \infty)$
3	$-2/3$
4	$100/27$
5	y-3 = (-1/5)(x – 1) or y = (-1/5)x + 16/5
6	$(-\infty, -3)$
7	2
8	$a(t) = \frac{1}{3}(t^2 - 6t)^{-2/3}(2t - 6)$
9	$\left(\frac{1}{2}, \frac{1}{2}\right)$
Tie-Break Round (Answers)—Played at Buzzers	
TB1	$\frac{2}{3}(x+5)^{3/2} - 10\sqrt{x+5} + C$
TB2	1358/27
TB3	0
TB4	470/13 mph
TB5	$f'(x) = 5\sin^4(e^{3x})\cos(e^{3x})e^{3x} * 3 = 15e^{3x}\sin^4(e^{3x})\cos(e^{3x})$