-ch. 3 review ANS (p. 1)

① $h(x)=\sqrt{x+3}=(x+3)^{1/2}$

$h'(x)=\frac{1}{2}(x+3)^{-1/2}(1)=\frac{1}{2\sqrt{x+3}}$

$m_T=h'(6)=\frac{1}{2\sqrt{9}}=\frac{1}{2\cdot 3}=\frac{1}{6}$

$\boxed{m_T=1/6}$ (A)

② $y(x)=-2\sin x$

$y'(x)=-2\cos x$ (B)

③ $\frac{d}{dx}\left[\overset{u}{(-3x^2-4x+3)}\overset{v}{(\sin x)}\right]$

$=(-3x^2-4x+3)(\cos x)+(\sin x)(-6x-4)$

$=\boxed{-3x^2\cos x-4x\cos x+3\cos x-6x\sin x-4\sin x}$

(B)

④ $y^4=-3x^2+4x-4$

$4y^3\frac{dy}{dx}=-6x+4$

$\boxed{\frac{dy}{dx}=\frac{-6x+4}{4y^3}}$ (D)

⑤ $\underset{u}{2x^2}\underset{v}{y}+2y^3=4$

$2x^2\frac{dy}{dx}+y(4x)+6y^2\frac{dy}{dx}=0$ (C)

$(2x^2+6y^2)\frac{dy}{dx}=-4xy$

$\boxed{\frac{dy}{dx}=\frac{-4xy}{(2x^2+6y^2)}}$

 [redacted] - ch. 3 review ANS

⑥ $y^4 = 3x^2 - 3x - 5$ $\quad (-1,-1)$

$4y^3 \frac{dy}{dx} = 6x - 3$

$\frac{dy}{dx} = \frac{6x-3}{4y^3}$

$m_T = \frac{6(-1)-3}{4(-1)^3} = \frac{-9}{-4}$

$m_T = 9/4$

(SO NOT A or B)

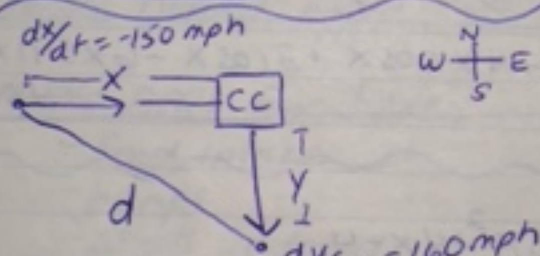
$y + 1 = \frac{9}{4}(x+1)$

$y + \frac{4}{4} = \frac{9}{4}x + \frac{9}{4}$

$\boxed{y = \frac{9}{4}x + 5/4}$

D

⑦ $dx/dt = -150$ mph




$dy/dt = 160$ mph

FIND $\frac{dd}{dt}$ when $T = .5$ hrs (AKA 30 min)

$x^2 + y^2 = d^2$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2d\frac{dd}{dT}$

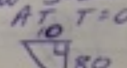
÷ everyone by 2

$x\frac{dx}{dt} + y\frac{dy}{dt} = d\frac{dd}{dT}$

$$\boxed{\frac{x(dx/dt) + y(dy/dt)}{d} = \frac{dd}{dt}}$$

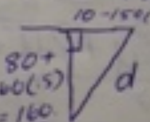
Now substitute

AT $T=0$




SO AT $T = .5$ $x = 10 - 150(.5) = -65$ mi

(IT'S passed through CC)

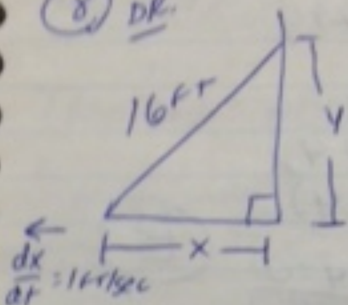



$$\frac{(-65)(-150) + (160)(160)}{\sqrt{(-65)^2 + (160)^2}} = \frac{dd}{dt}$$

$= 204.691$ mph

E

(8) DR:

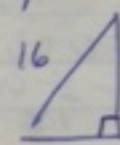



E: $x^2+y^2=16^2$

D: $2x\frac{dx}{dt}+2y\frac{dy}{dt}=0$

$\frac{dy}{dt}=\frac{-x\,dx/dt}{y}$

S: AT T=3




$y=\sqrt{16^2-25}$

$y=\sqrt{256-25}$

$y=\sqrt{231}$

$x=2+3=5$

initial x, change from 3sec

$\frac{dy}{dt}=\frac{-5(1)}{\sqrt{231}}\approx\boxed{-.329\ \text{ft/sec}}$ (D)

(9) $y(x)=|5x+2|$ which looks like A "Vee" $\vee$

the only critical point will occur at the POINT of the V (since the fxn won't be differentiable where it is pointy... so y' DNE)... which is when $5x+2=0$ $\boxed{x=-2/5}$

(A)

(10) $b(x)=\frac{x^2}{4x-2}$

$b'=\frac{(4x-2)(2x)-(x^2)(4)}{(4x-2)^2}=\frac{8x^2-4x-4x^2}{(4x-2)^2}$

$b'=\frac{4x^2-4x}{(4x-2)^2}=\frac{4x(x-1)}{(4x-2)^2}$

(D)

| | | |
|---|---|---|
| $b'=0$ | $b'=0$ | $b'$ DNE |
| $4x=0$ | $x-1=0$ | $4x-2=0$ |
| $\boxed{x=0}$ | $\boxed{x=1}$ | $\boxed{x=1/2}$ |

(VA) is not actually a point on fxn so not a CRIT. PT

# - ch. 3 review ANS p. 4

(11) $K(x) = -2x^3 - 13x^2 - 8x + 19$

$K'(x) = -6x^2 - 26x - 8$

$= -2(3x^2 + 13x + 4)$

$-2(3x+1)(x+4)$

(B)

$K' = 0$

$-2 = 0$ lie　　$3x + 1 = 0$ $\boxed{x = -1/3}$　　$x + 4 = 0$ $\boxed{x = -4}$

---

(12) $u(x) = \dfrac{e^x}{x-2}$

$u'(x) = \dfrac{(x-2)e^x - e^x(1)}{(x-2)^2} = \dfrac{xe^x - 2e^x - e^x}{x^2 - 4x + 4}$

$$\boxed{u'(x) = \frac{xe^x - 3e^x}{x^2 - 4x + 4}}$$ (A)

---

(13) $T(x) = x^3 + 9x^2 + 24x - 2$

$T'(x) = 3x^2 + 18x + 24$

$= 3(x^2 + 6x + 8) = 3(x+2)(x+4)$

$T'(x) = 0$　　$x + 2 = 0$　　$x + 4 = 0$

$3 = 0$ lie　　$x = -2$　　$x = -4$

$-\infty$　$-4$　$-2$　$\infty$

| | | | |
|---|---|---|---|
| 3 | + | + | + |
| $x+4$ | − 0 | + | + |
| $x+2$ | − | − 0 | + |
| | + | − | + |
| | ↗ | ↘ | ↗ |

$\boxed{\begin{array}{l} \text{INC: } (-\infty, -4) \cup (-2, \infty) \\ \text{Dec } (-4, -2) \end{array}}$ (A)

– ch. 3 review ANS p. 5

(14) $h(-1)=0$, $(hp)'(-1)=44$, $h'(-1)=11$

$(hp)'(x) = h(x)\cdot p'(x) + p(x)\cdot h'(x)$

$(hp)'(-1) = h(-1)\cdot p'(-1) + p(-1)\cdot h'(-1)$

given ↓ given ↓ ↓ given

$44 = 0 \cdot p'(-1) + p(-1)(11)$

$44 = 11 \cdot p(-1)$

$\frac{44}{11} = \boxed{4 = p(-1)}$

(C)

(15) $Q(x) = x^3 - 3x^2 - 24x - 4$

$Q'(x) = 3x^2 - 6x - 24$

$= 3(x^2 - 2x - 8) = 3(x-4)(x+2)$

$Q'(x) = 0$

$3 = 0$ lie   $x-4=0$, $x=4$   $x+2=0$, $x=-2$

| | $-\infty$ to $-2$ | $-2$ to $4$ | $4$ to $\infty$ |
|---|---|---|---|
| 3 | + | + | + |
| $x-4$ | − | − (0 at 4) | + |
| $x+2$ | − (0 at −2) | + | + |
| | ⊕ | ⊖ | ⊕ |

MAX, MIN

$\boxed{\text{MAX @ } x = -2 \quad \text{MIN @ } x = 4}$

(C)

(16) $b(x) = -x^3 + 12x + 4$

$b'(x) = -3x^2 + 12$

$b''(x) = -6x$

$b''(x) = 0$

$-6x = 0$

$x = 0$

| | $-\infty$ to $0$ | $0$ to $\infty$ |
|---|---|---|
| $-6x$ | + (0 at 0) | − |
| | ∪ | ∩ |

$\boxed{\text{CON. UP } (-\infty, 0) \quad \text{CON. DOWN } (0, \infty)}$

(B)

(17) $d(x) = 3x^5 - 15x^4 + 30x^3 - 30x^2 + 4x - 5$

$d'(x) = 15x^4 - 60x^3 + 90x^2 - 60x + 4$

$d''(x) = 60x^3 - 180x^2 + 180x - 60$

$= 60(x^3 - 3x^2 + 3x - 1)$

1 1 1 2 1 1 3 3 1

$d''(x) = 60(x-1)^3$

$d'' = 0$

$60 = 0$ lie

$(x-1)^3 = 0$

$x - 1 = 0$

$x = 1$

happens odd # of times so write it once

| | $-\infty$ | 1 | $\infty$ |
|---|---|---|---|
| 60 | + | | + |
| $(x-1)$ | − | 0 | + |
| | ∩ | | ∪ |

POI

$\boxed{x = 1}$ (A)

---

(18) $a(x) = -x^3 - 6x^2 - 12x + 3$

$a'(x) = -3x^2 - 12x - 12$

$= -3(x^2 + 4x + 4)$

$a'(x) = -3(x+2)^2$

$a''(x) = -6x - 12$

CRIT PTS

$a' = 0$

$-3 = 0$ lie

$(x+2)^2 = 0$

$\boxed{x = -2}$

2ND Derivative test says check critical pt in 2ND derivative... if $A''(-2) \oplus$ then ∪ so min

if $A''(-2) \ominus$ then ∩ so max

if $A''(-2) = 0$ it's inconclusive

So...

$A''(x) = -6x - 12$

$A''(-2) = -6(-2) - 12 = 12 - 12 = 0$

$\boxed{\text{INCONCLUSIVE}}$ (D)

Chapter 3 Review-Answer Key

| | | |
|---|---|---|
| 1. A | 7. E. (204.691 mph) | 13. A |
| 2. B | 8. D | 14. C |
| 3. B | 9. A | 15. C |
| 4. D | 10. D | 16. B |
| 5. C | 11. B | 17. A |
| 6. D | 12. A | 18. D |

1996 AB1

(a) $x=-2$

$f'(x)$ changes from positive to negative at $x=-2$

or

$f$ is increasing to the left of $x=-2$ and decreasing to the right of $x=-2$

(b) $x=4$

$f'(x)$ changes from negative to positive at $x=4$

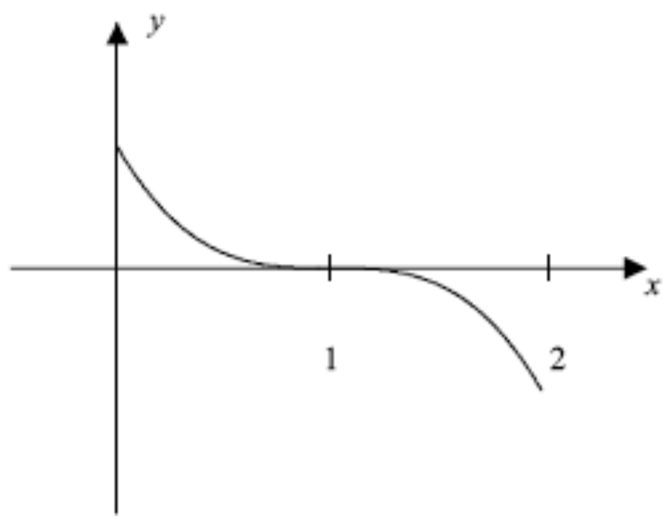
or

$f$ is decreasing to the left of $x=4$ and increasing to the right of $x=4$

(c) $(-1,1)$ and $(3,5)$

$f'$ is increasing on these intervals.

(d)



Chapter 3 Review-Answer Key

1971 AB3

$f(x) = \cos^2 x + 2\cos x$ in $[0, 2\pi)$

(a) $f(x) = \cos x(\cos x + 2) = 0$ when $\cos x = 0$. So $x = \dfrac{\pi}{2}$ or $x = \dfrac{3\pi}{2}$.

(b) $f'(x) = -2\cos x \sin x - 2\sin x = -2\sin x(1 + \cos x)$
$f'(x) = 0$ when $x = 0$, $\pi$ (and $2\pi$).

Possible justifications for the location of the minimum:

| | |
|---|---|
| First derivative test:<br>$f'(x) < 0$ for $0 < x < \pi$ so the graph is decreasing on this interval.<br>$f'(x) > 0$ for $\pi < x < 2\pi$ so the graph is increasing on this interval.<br><br>Therefore there is a minimum at $x = \pi$. | Second derivative test:<br>$f''(x) = -2\cos^2 x + 2\sin^2 x - 2\cos x$<br>$= 2(1 - 2\cos x)(1 + \cos x)$<br>$f''(\pi) = 0$ so provides no conclusion. But $f''(x) > 0$ for $x$ just less than $\pi$ and just greater than $\pi$, thus the graph is concave up in an interval containing $x = \pi$, so $x = \pi$ gives a local minimum. Since it is the only interior critical point, it must be the location of the absolute minimum. |
| Test the critical points:<br>$f(0) = 3$<br>$f(\pi) = -1$<br>$f(2\pi) = 3$<br>Therefore there is a minimum at $x = \pi$ | Non-calculus reasoning:<br>$f(x) = (\cos x + 1)^2 - 1$<br>Because of the square, the minimum will occur when $\cos x + 1 = 0$, i.e. when $x = \pi$. |

(c) $f''(x) = 2(1 - 2\cos x)(1 + \cos x)$

$f''(x) = 0$ at $x = \dfrac{\pi}{3}$ and $x = \dfrac{5\pi}{3}$.

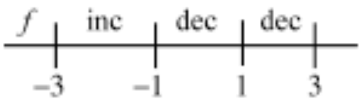
$f''(x) > 0$ for $\dfrac{\pi}{3} < x < \dfrac{5\pi}{3}$. Therefore the graph is concave up for $\dfrac{\pi}{3} < x < \dfrac{5\pi}{3}$.

Chapter 3 Review-Answer Key

1984 AB4/BC3

(a) The absolute maximum occurs at $x = -1$ because $f$ is increasing on the interval $[-3, -1]$ and decreasing on the interval $[-1, 3]$.

or




The absolute minimum must occur at $x = 1$ (the other critical point) or at an endpoint. However, $f$ is decreasing on the interval $[-1, 3]$. Therefore the absolute minimum is at an endpoint. Since $f(-3) = 4 > 1 = f(3)$, the absolute minimum is at $x = 3$.

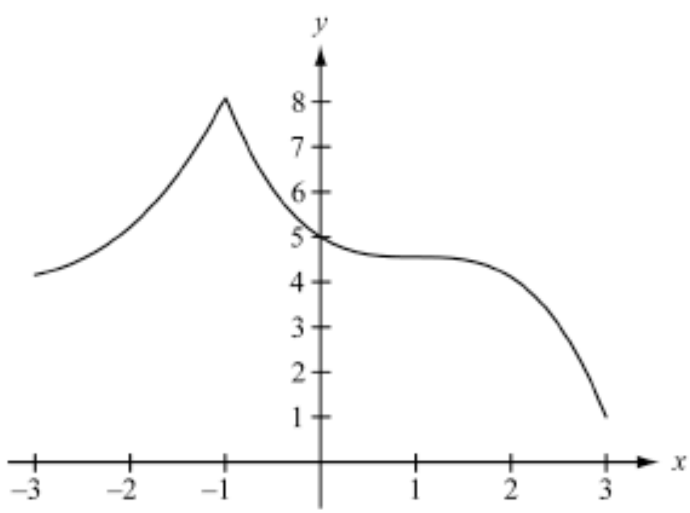
(b) There is an inflection point at $x = 1$ because:

the graph of $f$ changes from concave up to concave down at $x = 1$

or

$f''$ changes sign from positive to negative at $x = 1$

(c) This is one possibility:



S.Hogan

Chapter 3 Review-Answer Key

1994 AB1

(a) $f'(x) = 12x^3 + 3x^2 - 42x$

$f'(2) = 24$

$y + 28 = 24(x-2)$

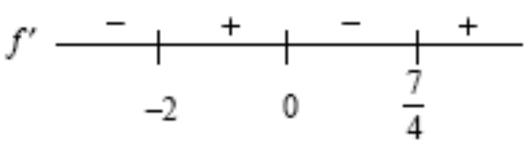
or $y = 24x - 76$

(b) $12x^3 + 3x^2 - 42x = 0$

$3x(4x^2 + x - 14) = 0$

$3x(4x-7)(x+2) = 0$

$x = 0, x = \frac{7}{4}, x = -2$




min must be at $-2$ or $\frac{7}{4}$.

$f(-2) = -44 \quad f\left(\frac{7}{4}\right) = -30.816$

Absolute min is $-44$

(c) $f''(x) = 36x^2 + 6x - 42$

$= 6(6x^2 + x - 7)$

$= 6(6x+7)(x-1)$

Zeros at $x = -\frac{7}{6}, \ x = 1$